Rasterization

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School of Data and Computer Science

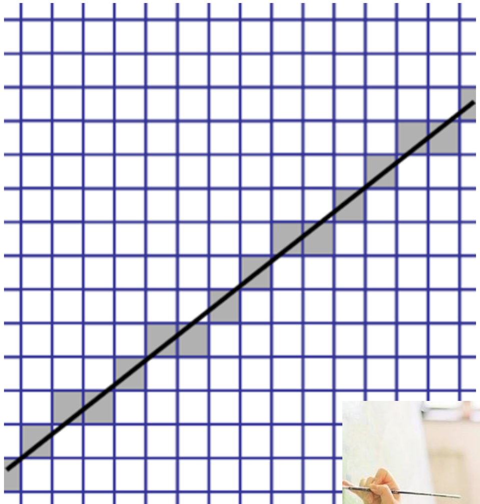
To make an image, we can...

Drawing

Photography

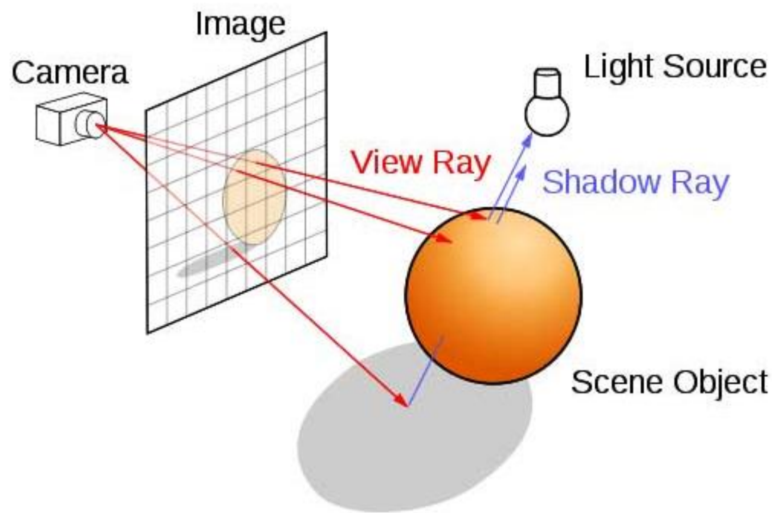
Two Ways to Render an Image

In CG, drawing is...



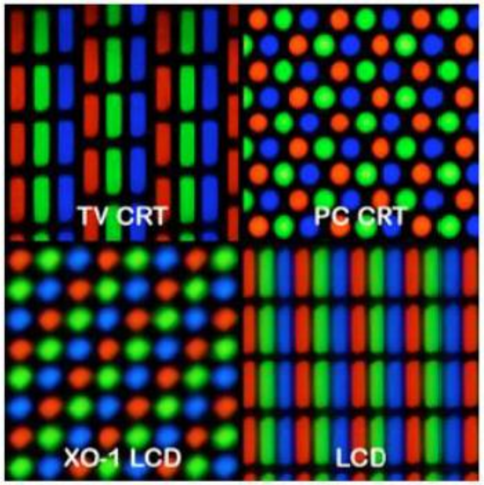
Rasterization

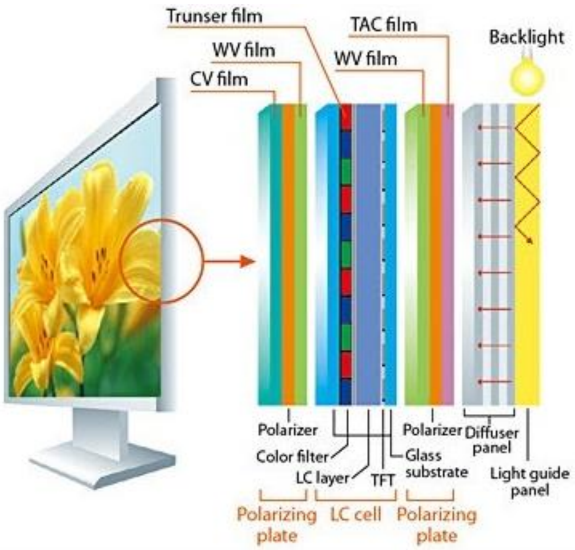
Photography is...



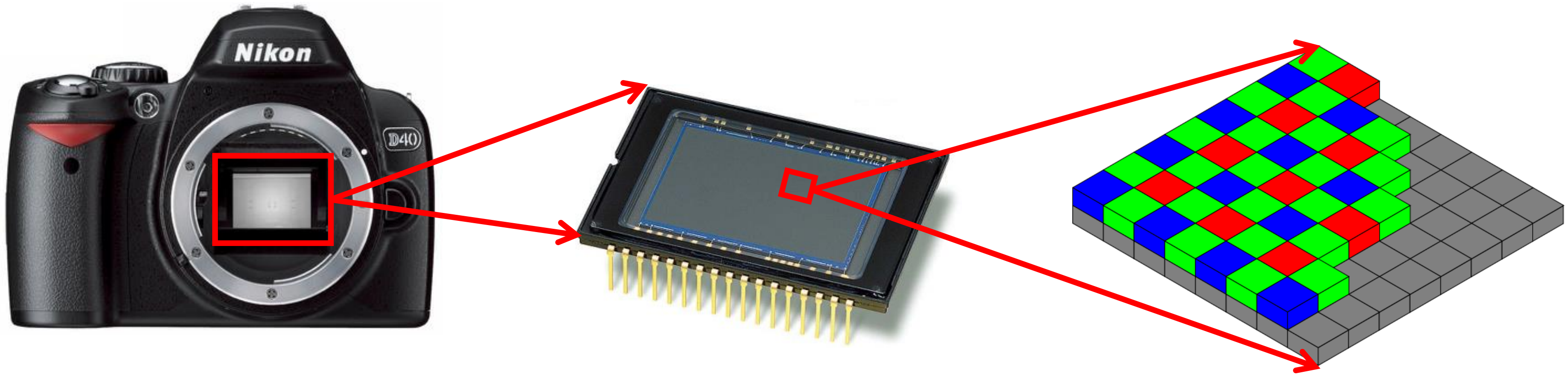
Ray Tracing

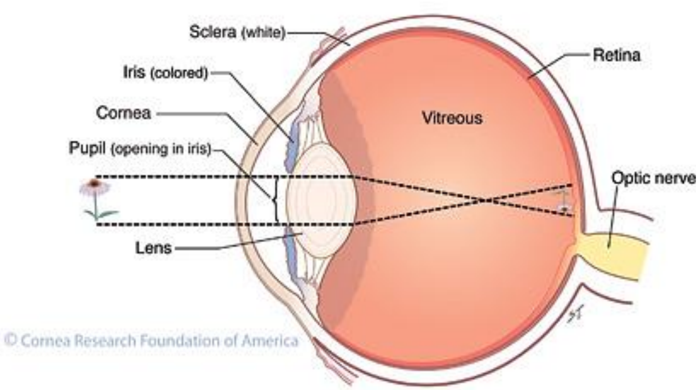
Screen


TV CRT
PC CRT
XO-1 LCD
LCD


Trunser film
WV film
CV film
TAC film
WV film
Backlight
Polarizer
Color filter
LC layer
TFT
Glass substrate
Polarizer
Diffuser panel
Light guide panel
Polarizing plate
LC cell
Polarizing plate

Sensors





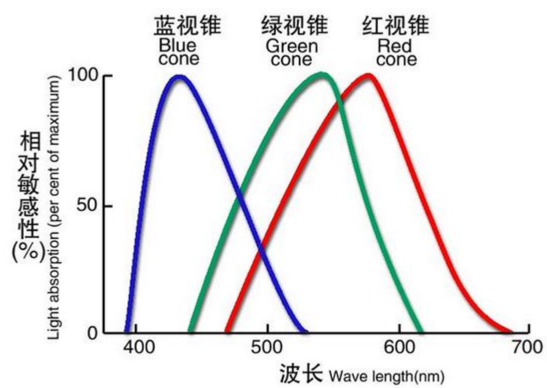
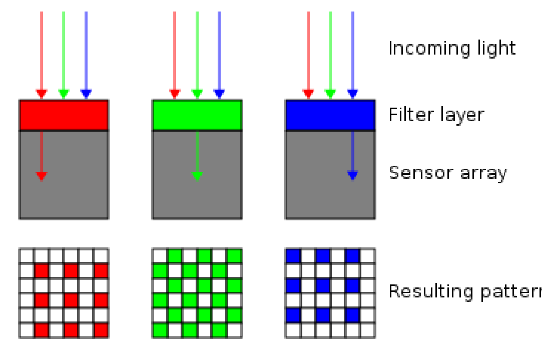


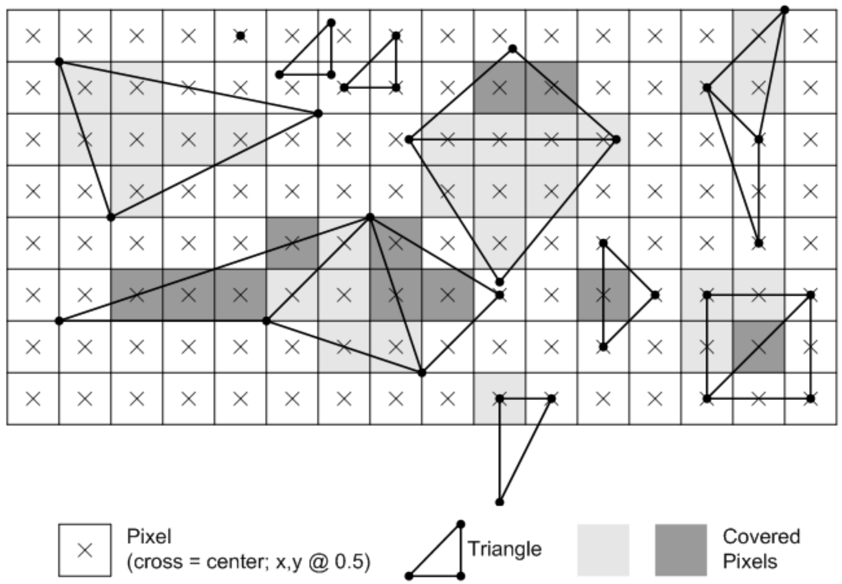
图 – 人视网膜中三种不同视锥细胞的光谱相对敏感性



真实物理世界没有颜色的概念，只有**频率**。颜色只是人的主观感受，不是物体的客观属性，物体只是在发射或反射电磁波。

Rasterization

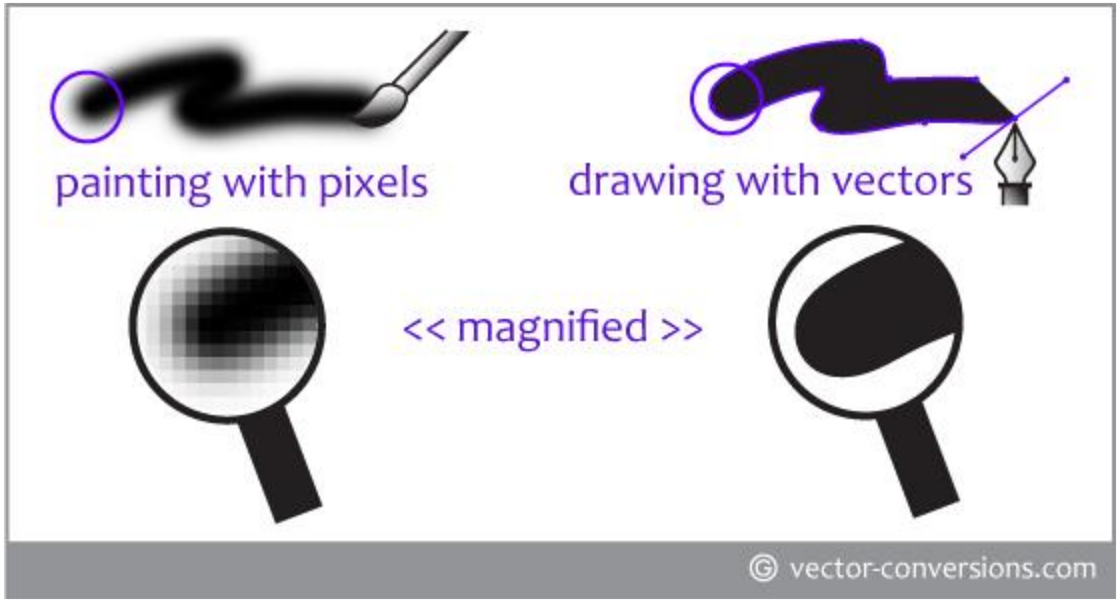
- The task of displaying a world modeled using primitives like lines, polygons, filled/patterned area, etc. can be carried out in two steps:
  - determine the pixels through which the primitive is visible,
  - determine the color value to be assigned to each such pixel



Raster Graphics Packages

- The efficiency of these steps forms the main criteria to determine **the performance of a display**
- The raster graphics package is typically **a collection of efficient algorithms** for scan converting (rasterization) of the display primitives
- High performance graphics workstations have most of these algorithms **implemented in hardware**
- Comparison of raster graphics editors：
- https://en.wikipedia.org/wiki/Comparison_of_raster_graphics_editors

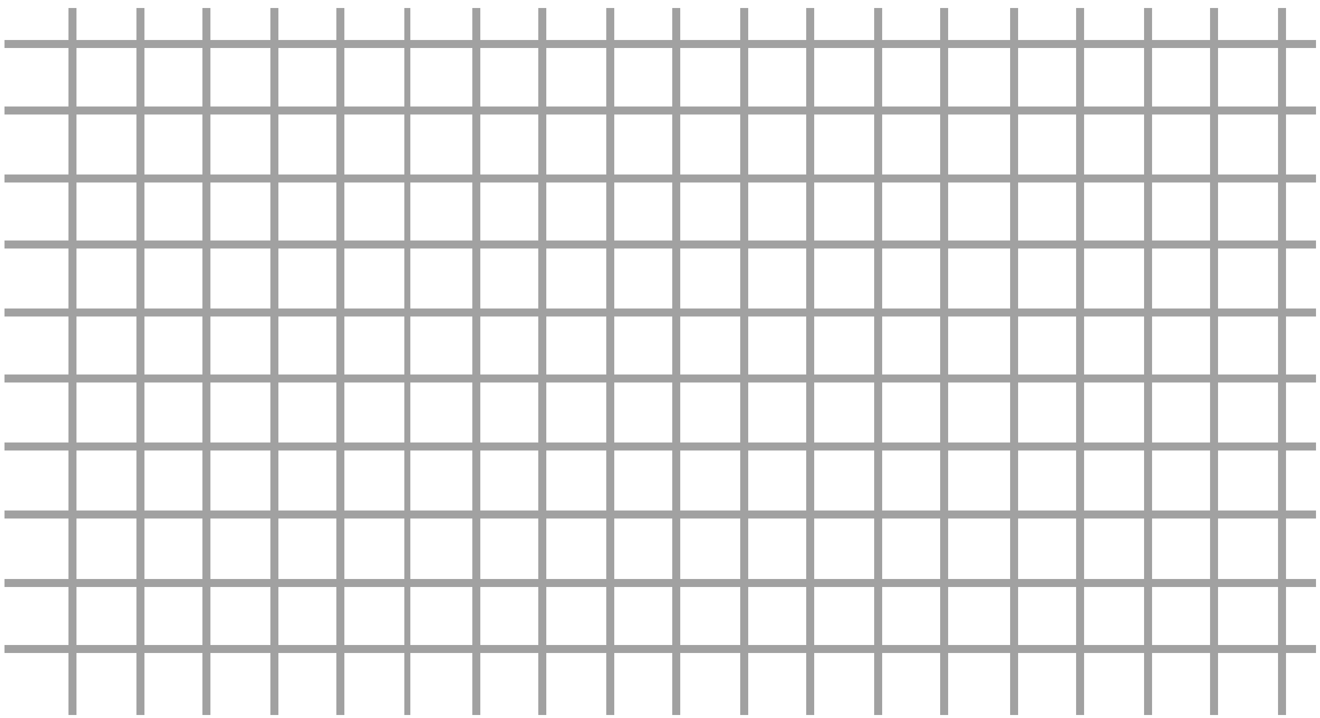
To convert **vector data** to raster format



Scan Conversion： Figure out which pixel should to shade.

Scan converting lines

start from (x_1, y_1) end at (x_2, y_2)



Scan converting lines

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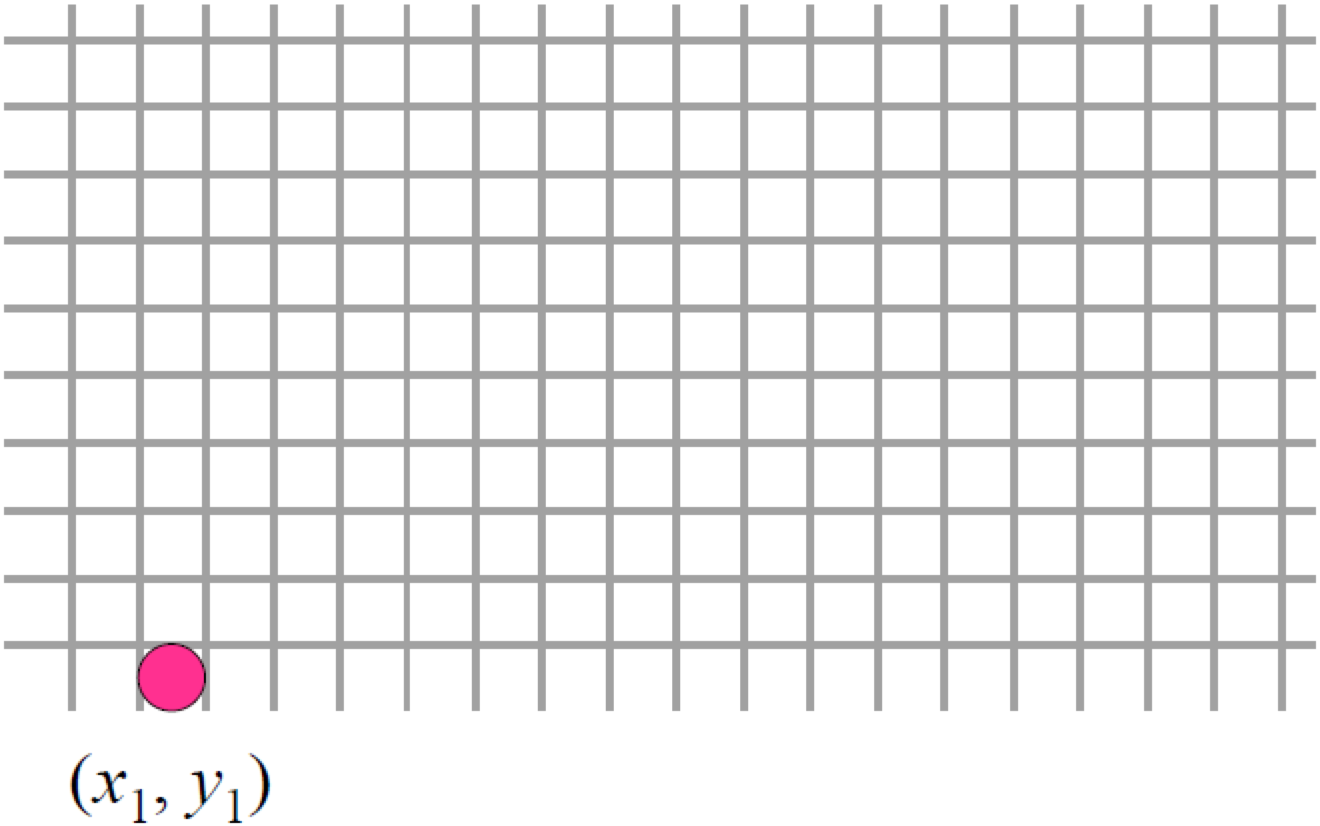
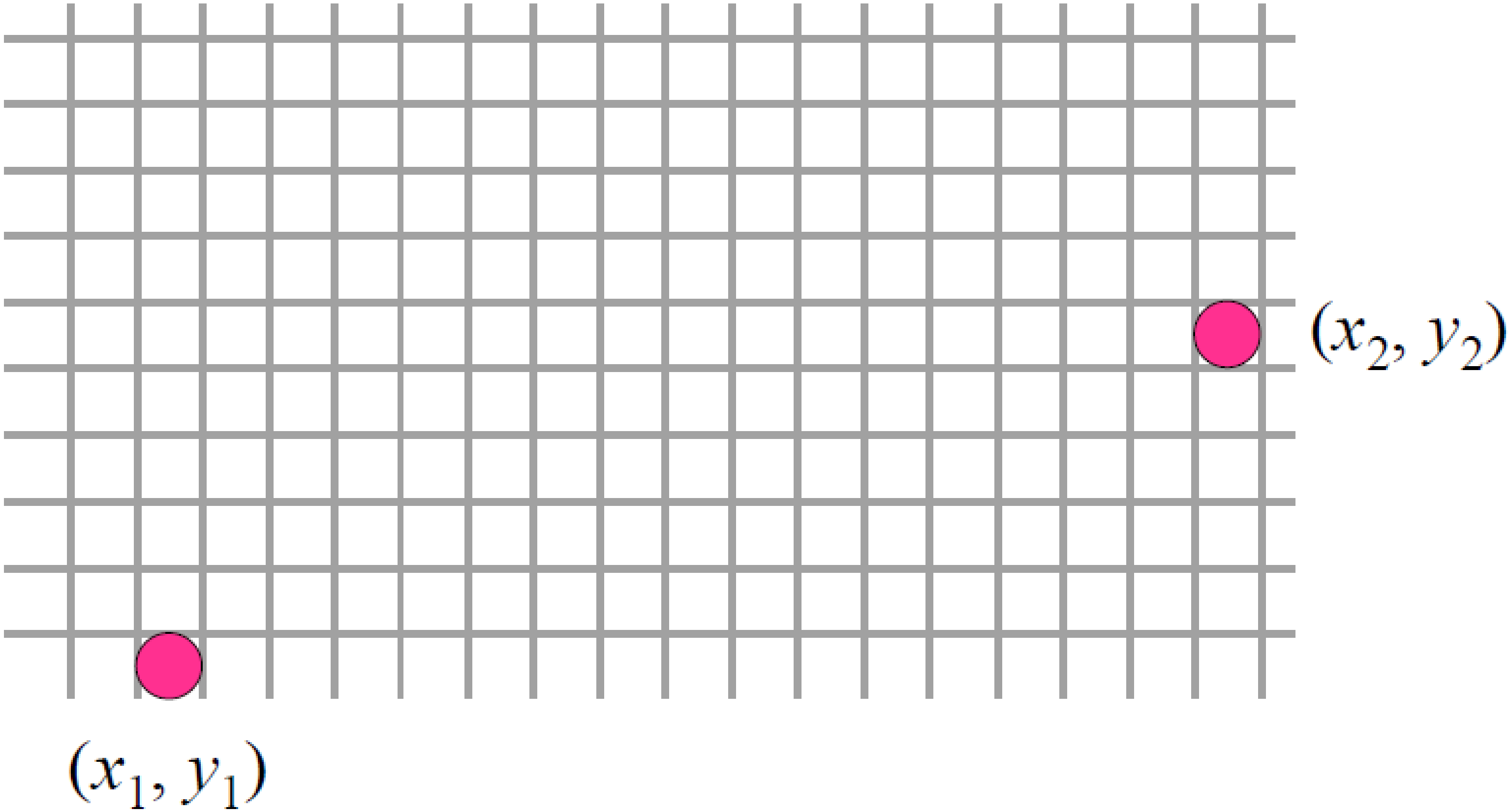


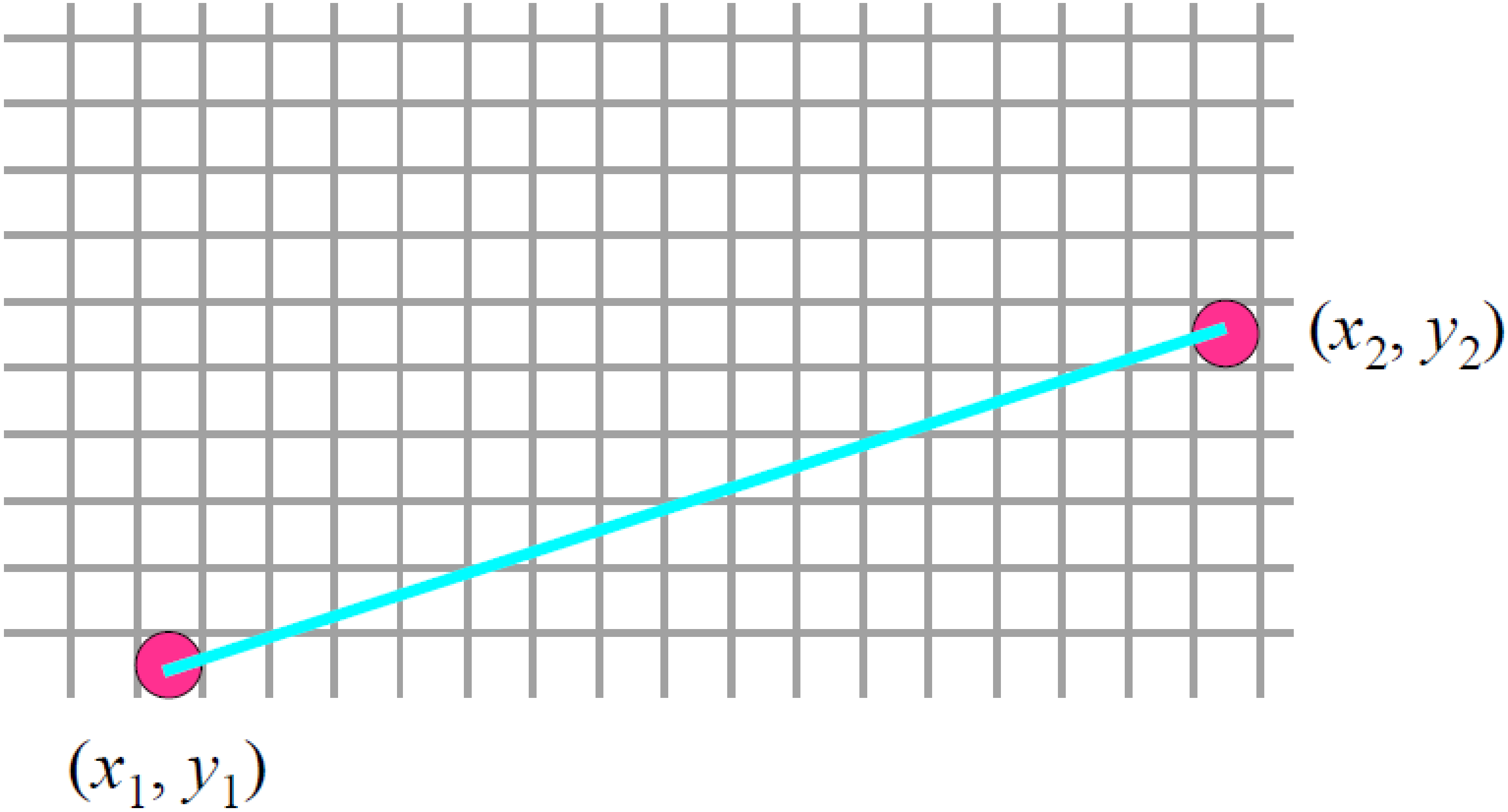
Scan converting lines

start from (x_1, y_1) end at (x_2, y_2)



Scan converting lines

start from (x_1, y_1) end at (x_2, y_2)



Scan converting lines

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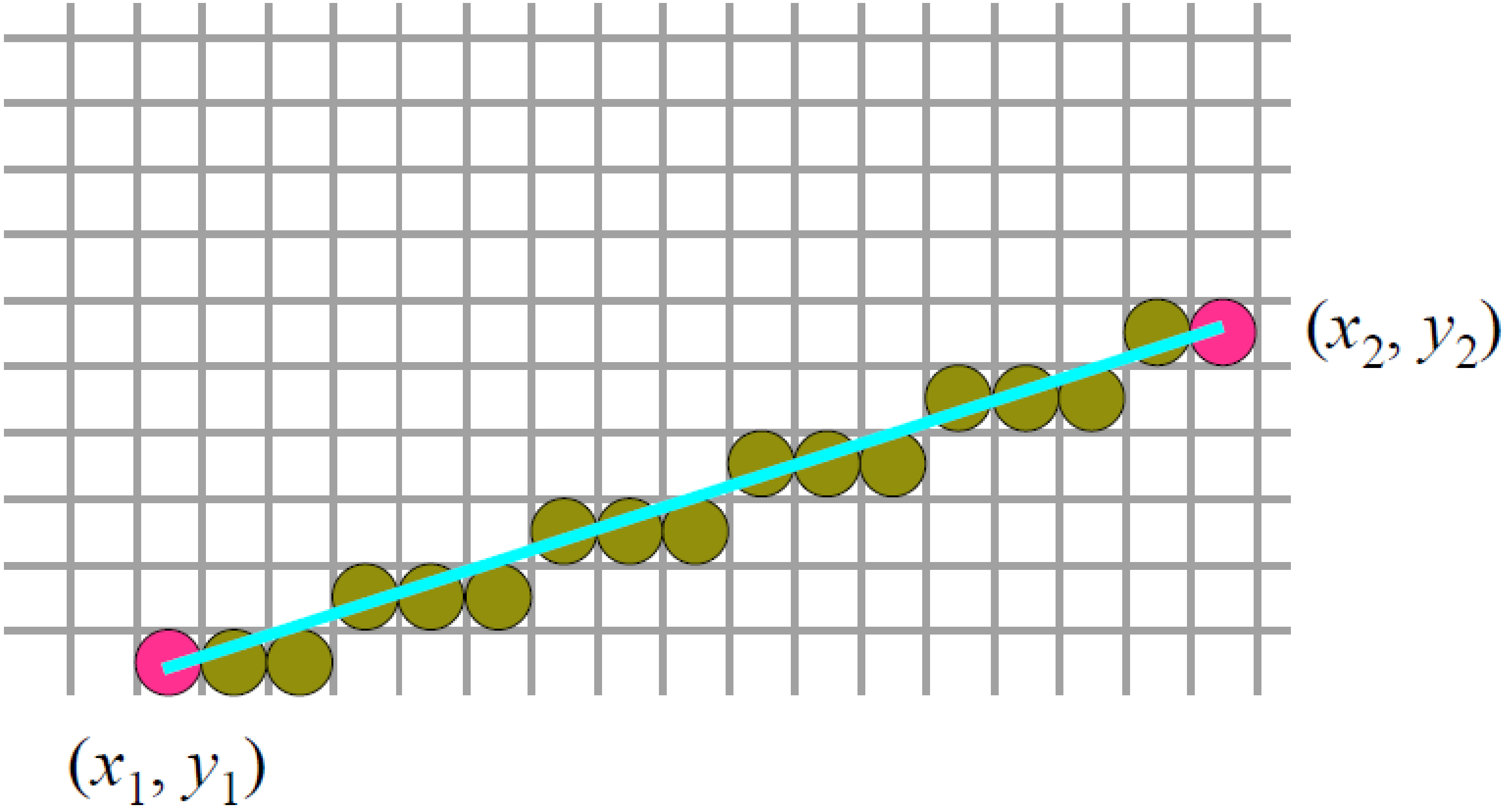


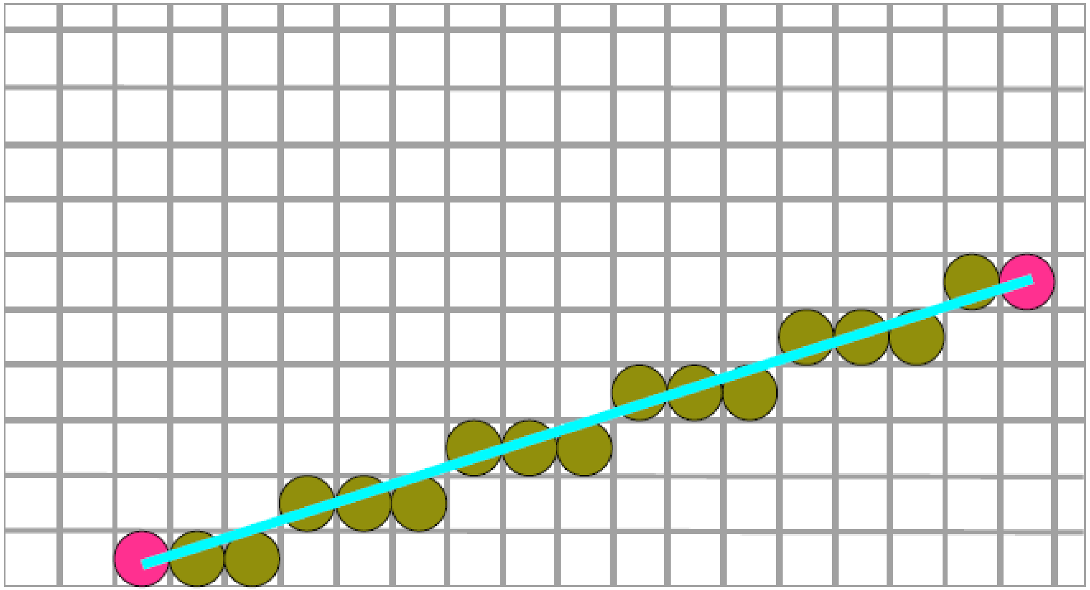
Scan converting lines

- Requirements
  - chosen pixels should lie as close to the ideal line as possible
  - the sequence of pixels should be as straight as possible
  - all lines should appear to be of constant brightness independent of their length and orientation
  - should start and end accurately
  - should be drawn as rapidly as possible
  - should be possible to draw lines with different width and line styles

Scan converting lines

Question 1: How ?

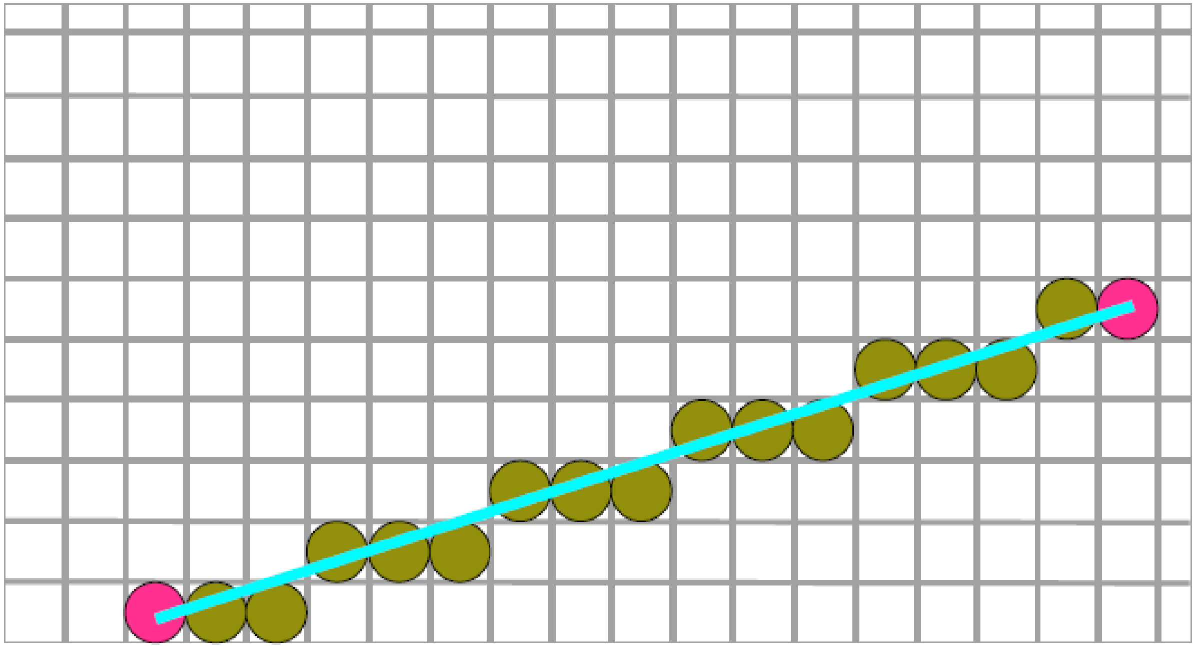
$(x_1, y_1), (x_2, y_2)$



Scan converting lines

Question 1: How ?

$(x_1, y_1), (x_2, y_2)$



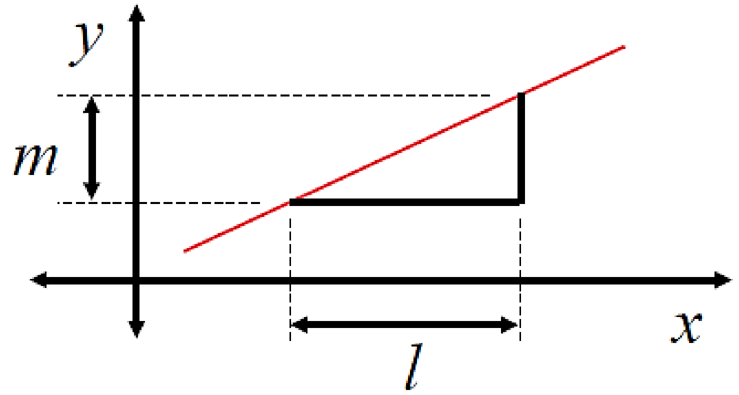
$y=mx+b$

$x_1+1 \Rightarrow y=?$, rounding

$x_1+2 \Rightarrow y=?$, rounding → $x_1+i \Rightarrow y=?$, rounding

Equation of Line

- Equation of a line is $y - m \cdot x + c = 0$
- For a line segment joining points
- $P(x_1, y_1)$ and $Q(x_2, y_2)$ *slope* $m = \dfrac{y2 - y1}{x2 - x1} = \dfrac{\Delta y}{\Delta x}$
- Slope m means that for every unit increment in x the increment in y is m units



Digital Differential Analyzer (DDA, 数值微分法)

- We consider the line in the first octant. Other cases can be easily derived.
- Uses differential equation of the line

$$y_i = mx_i + c$$

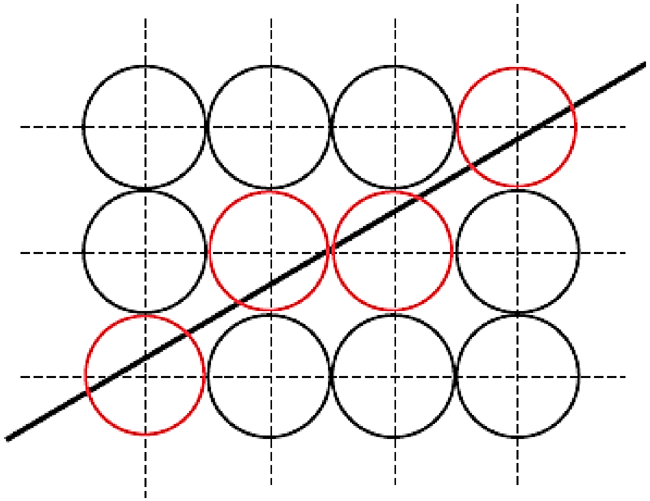
$$where, \quad m = \frac{y2 - y1}{x2 - x1}$$

- Incrementing X-coordinate by 1

$$x_i = x_{i\_prev} + 1$$

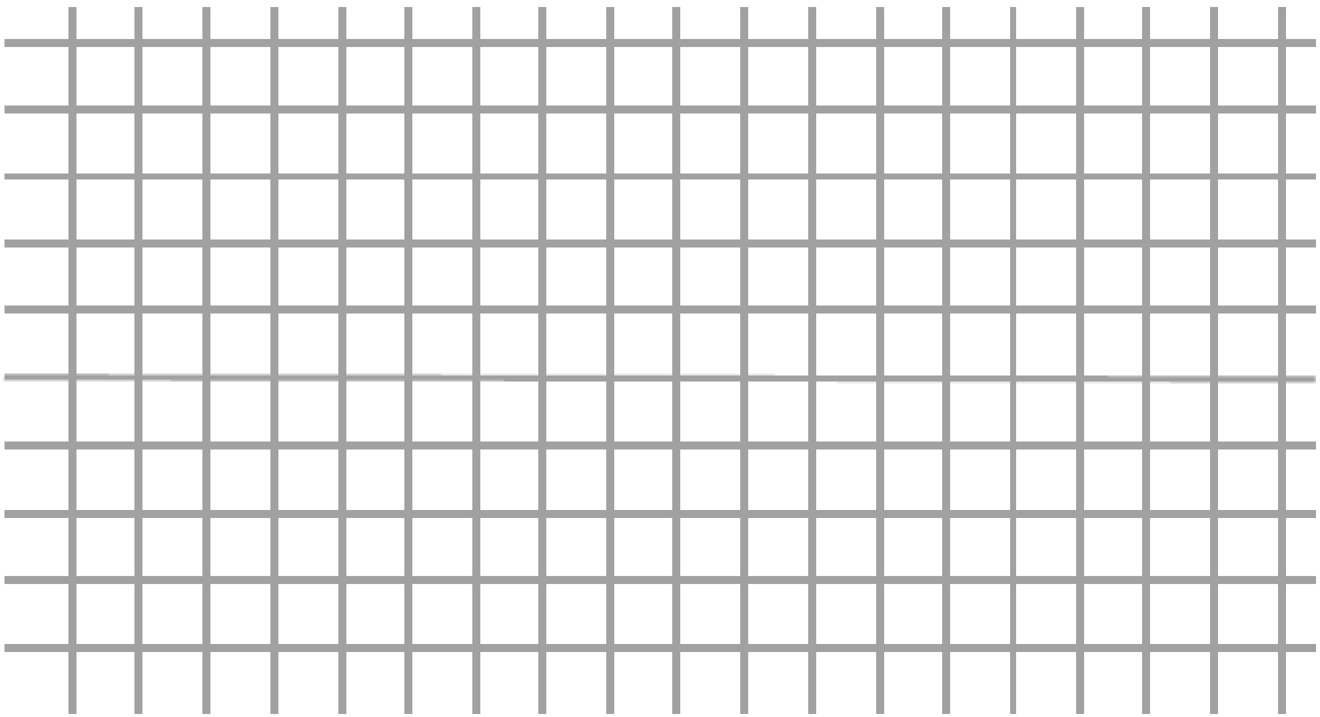
$$y_i = y_{i\_prev} + \mathrm{m}$$

- Illuminate the pixel $[x_i, round(y_i)]$



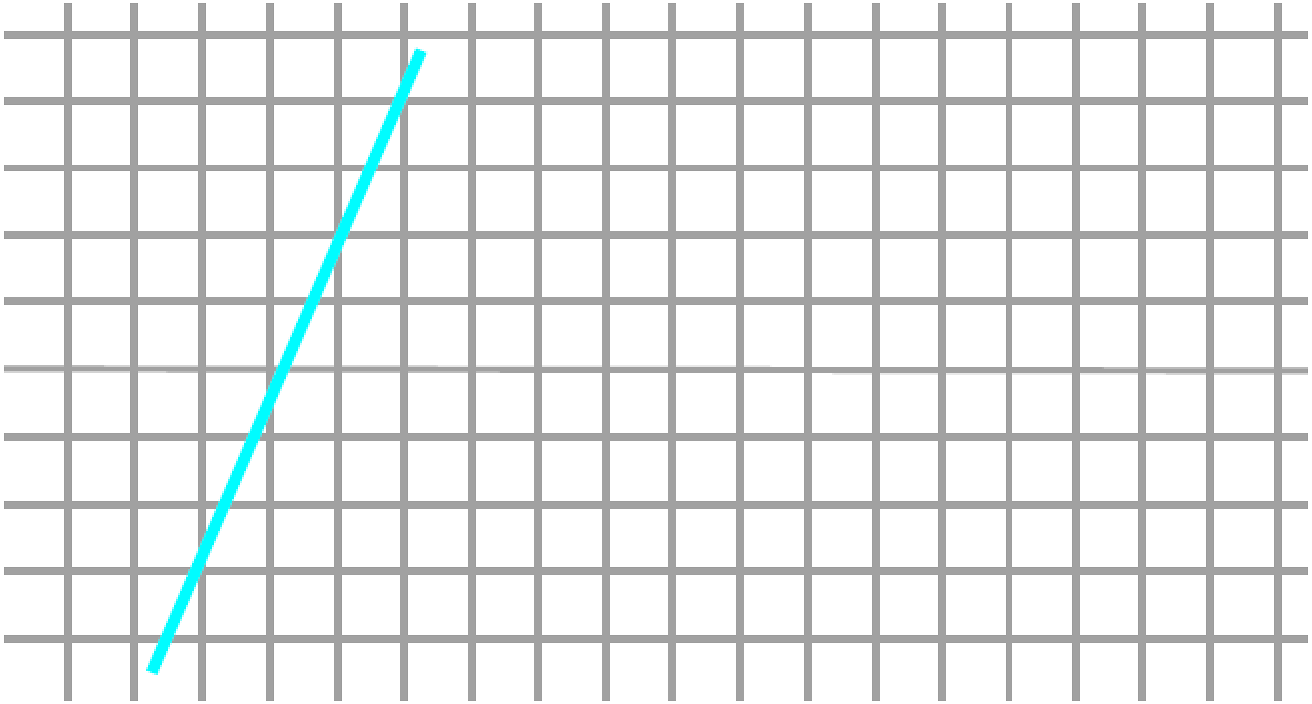
Digital Differential Analyzer (DDA, 数值微分法)

If $\triangle x < \triangle y$



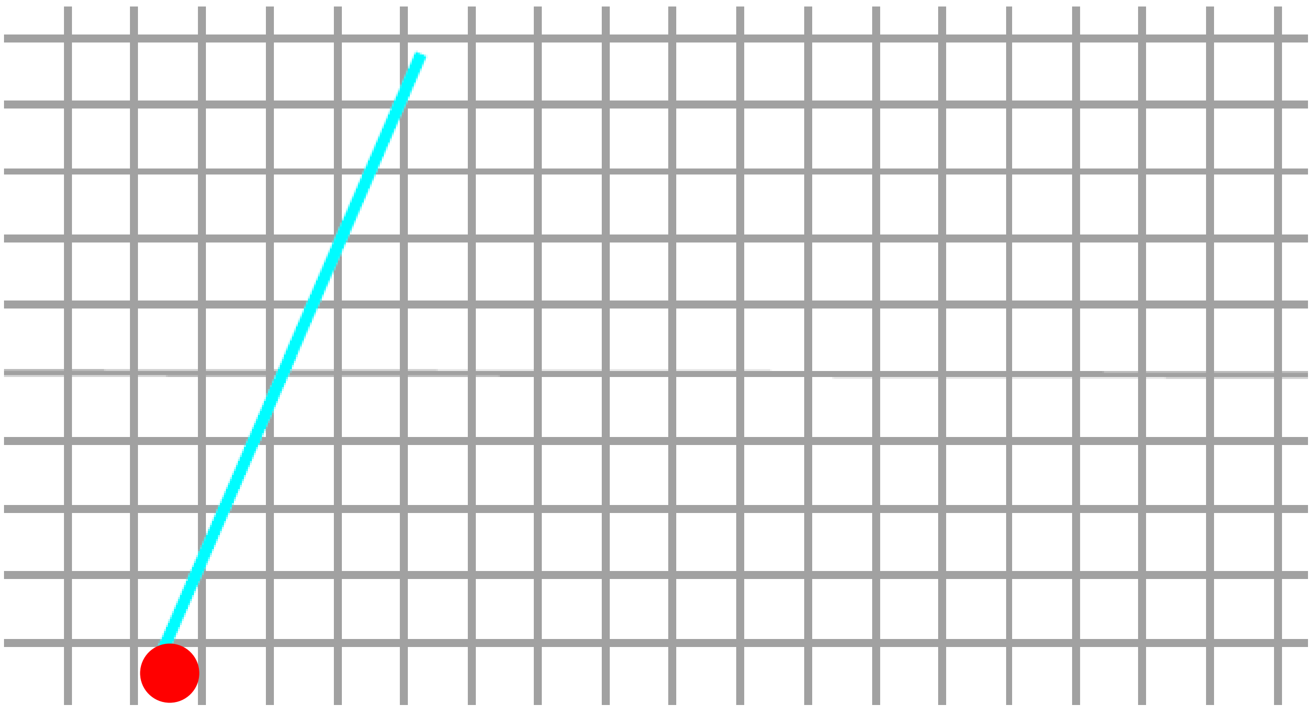
Digital Differential Analyzer (DDA, 数值微分法)

If $\triangle x < \triangle y$



Digital Differential Analyzer (DDA)

If $\triangle x < \triangle y$



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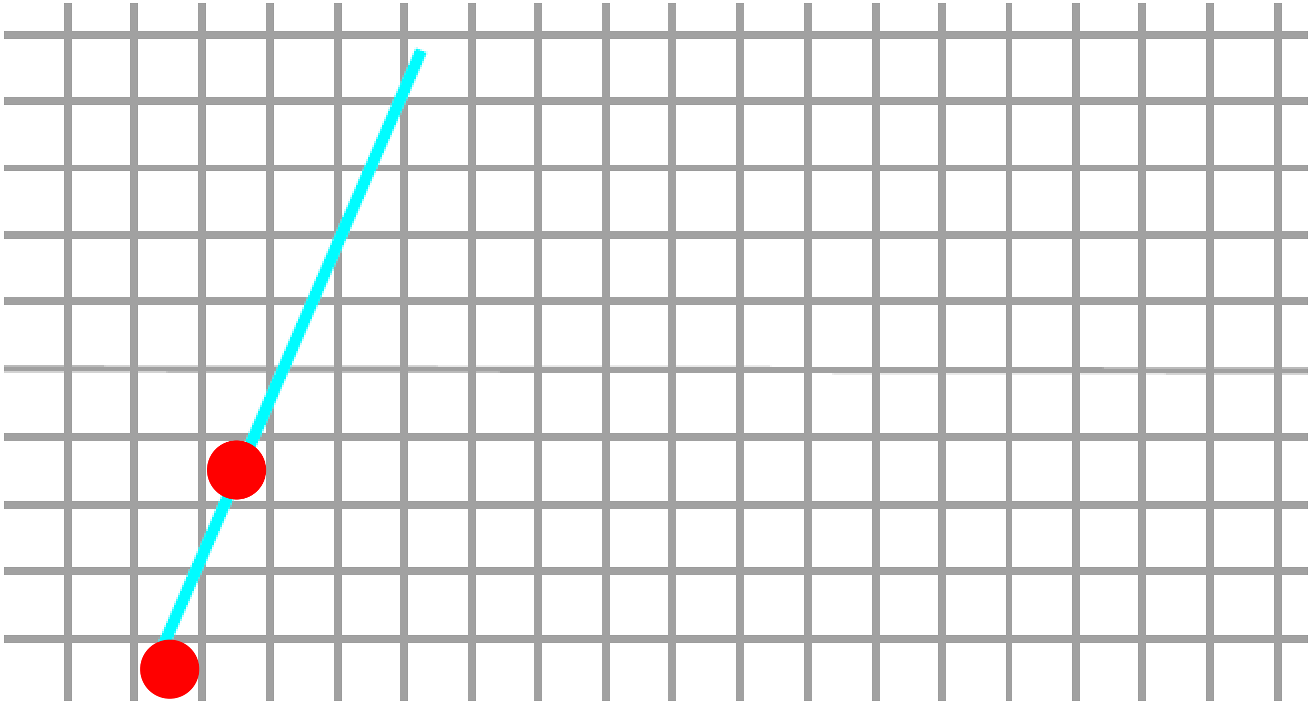

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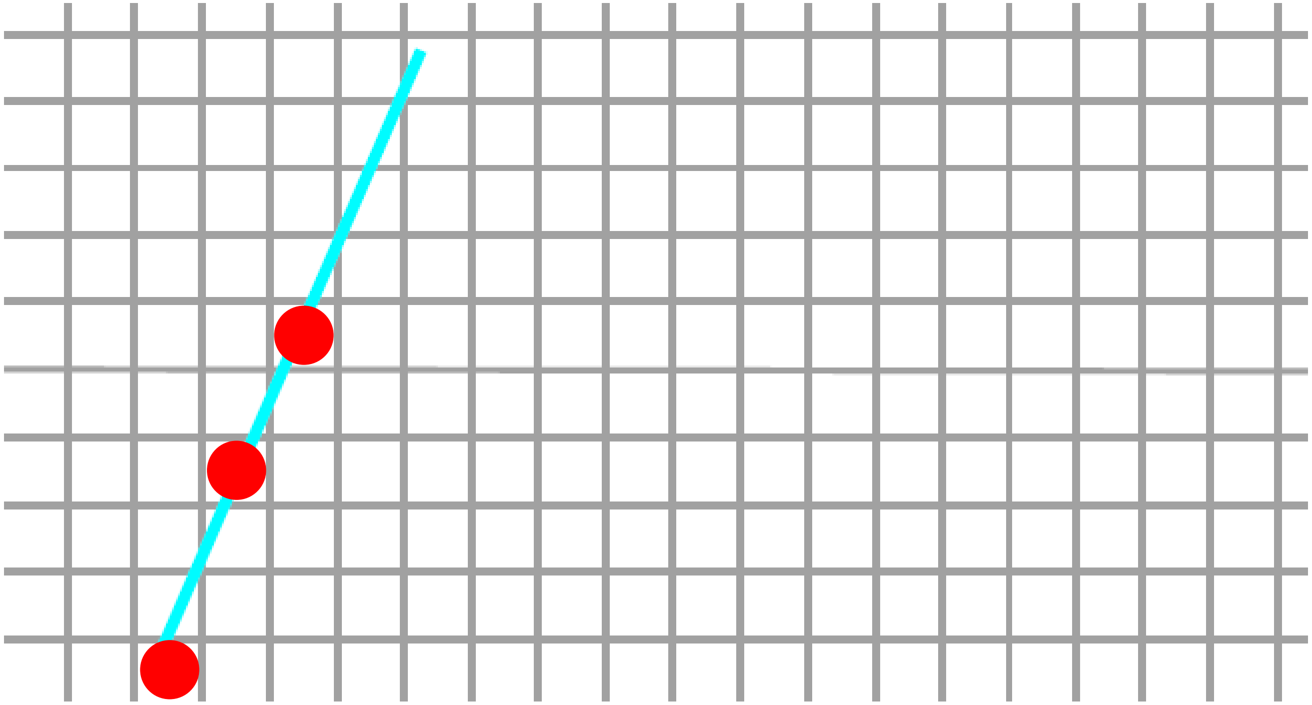

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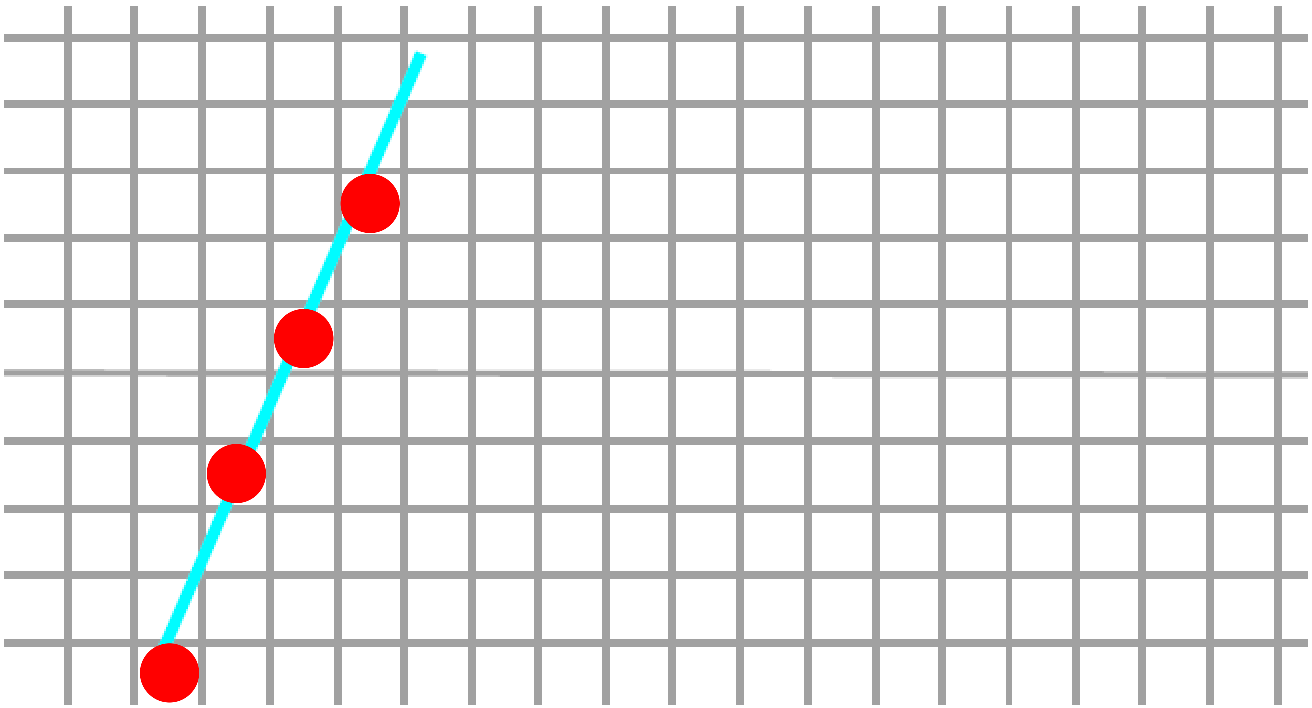
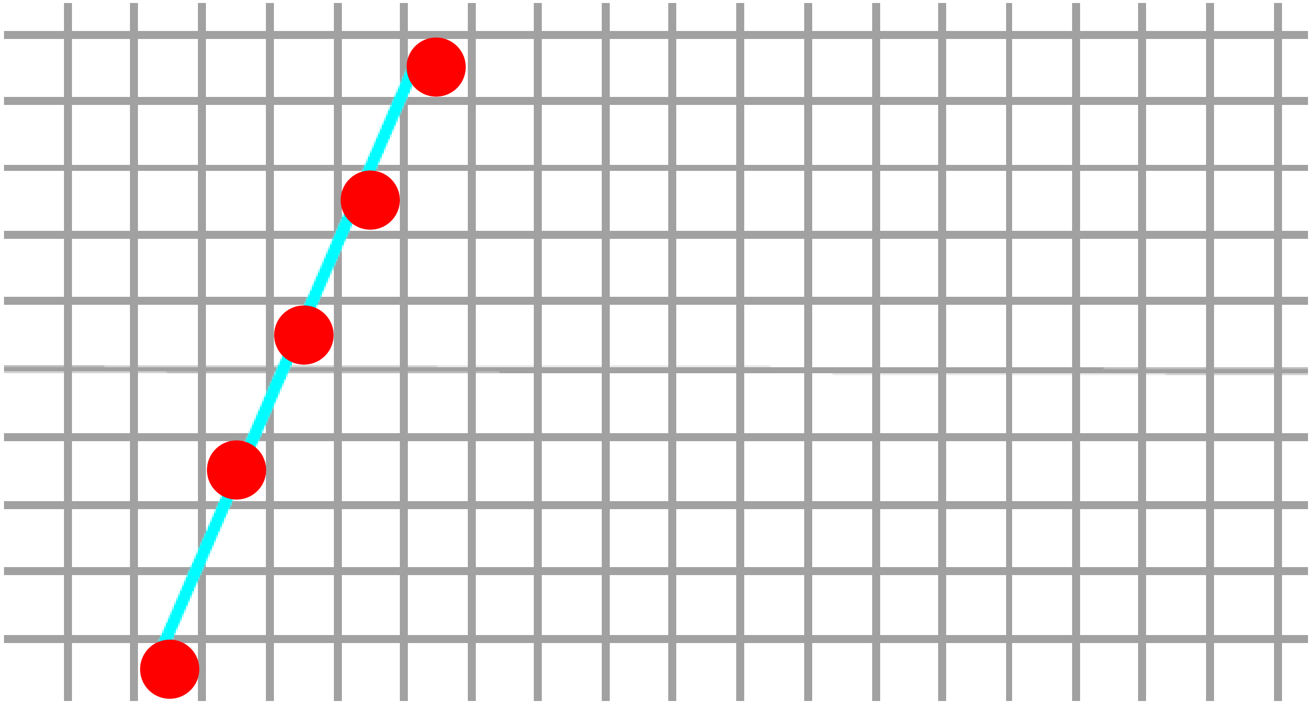


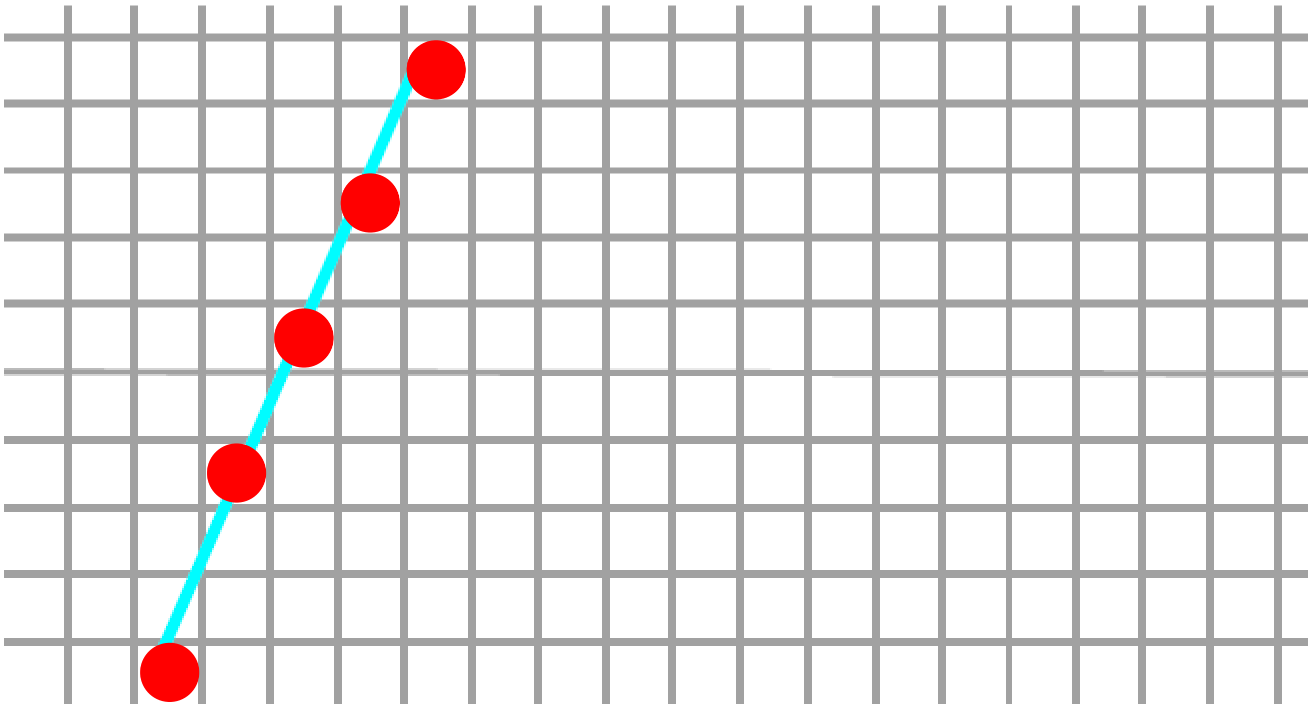
Digital Differential Analyzer (DDA)

If $\triangle x < \triangle y$



Digital Differential Analyzer (DDA)

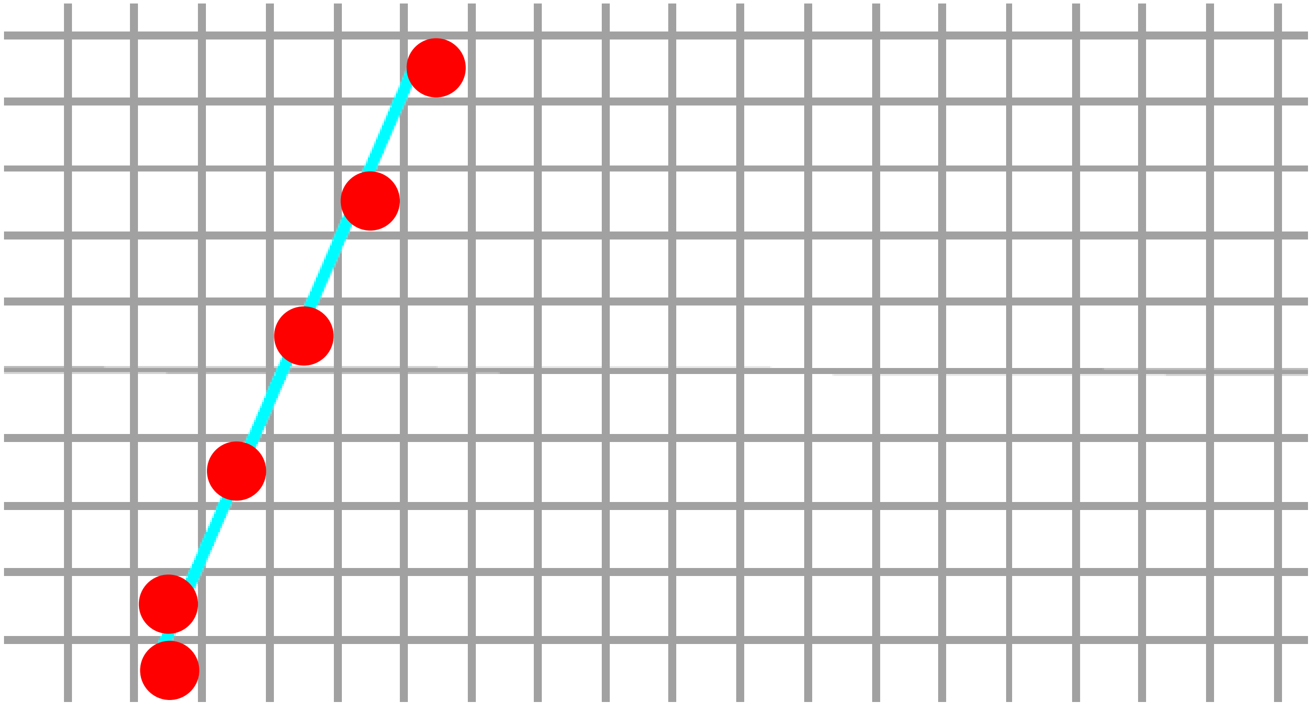
If $\triangle x < \triangle y$



y += 1, x += 1/m

Digital Differential Analyzer (DDA)

If $\triangle x < \triangle y$



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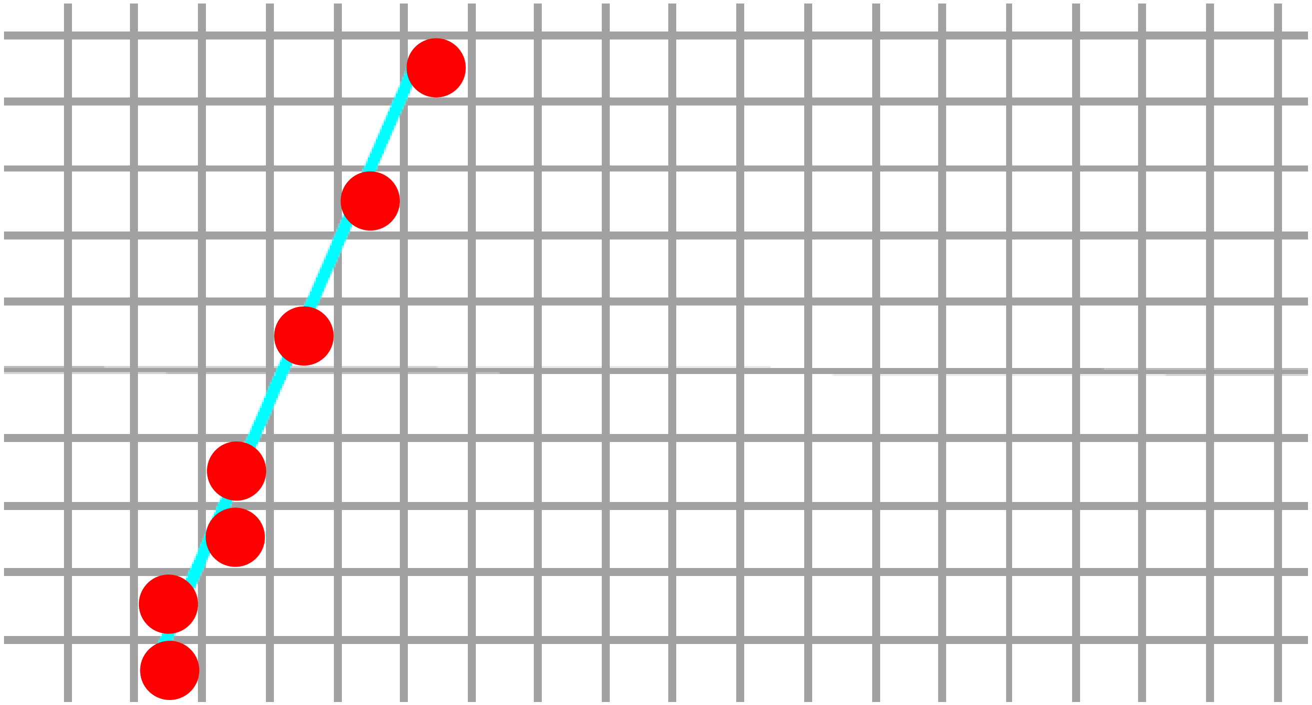
Digital Differential Analyzer (DDA)

If $\triangle x < \triangle y$



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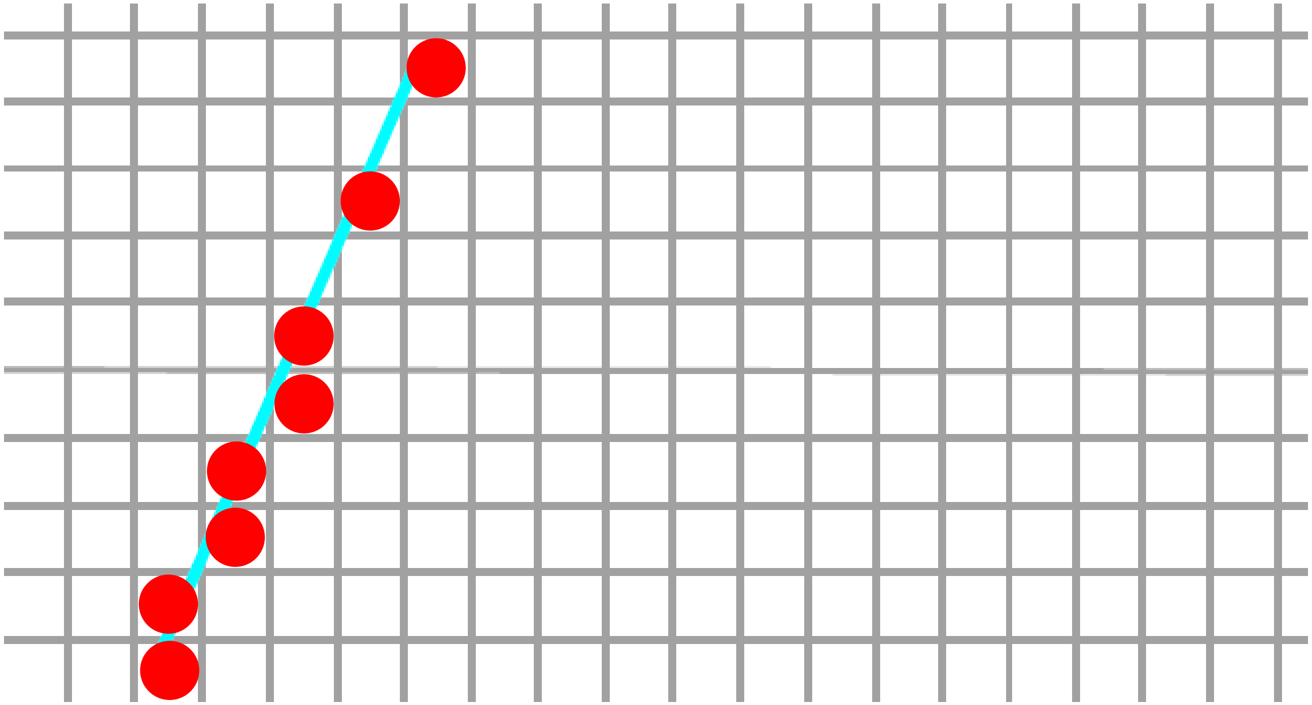
Digital Differential Analyzer (DDA)

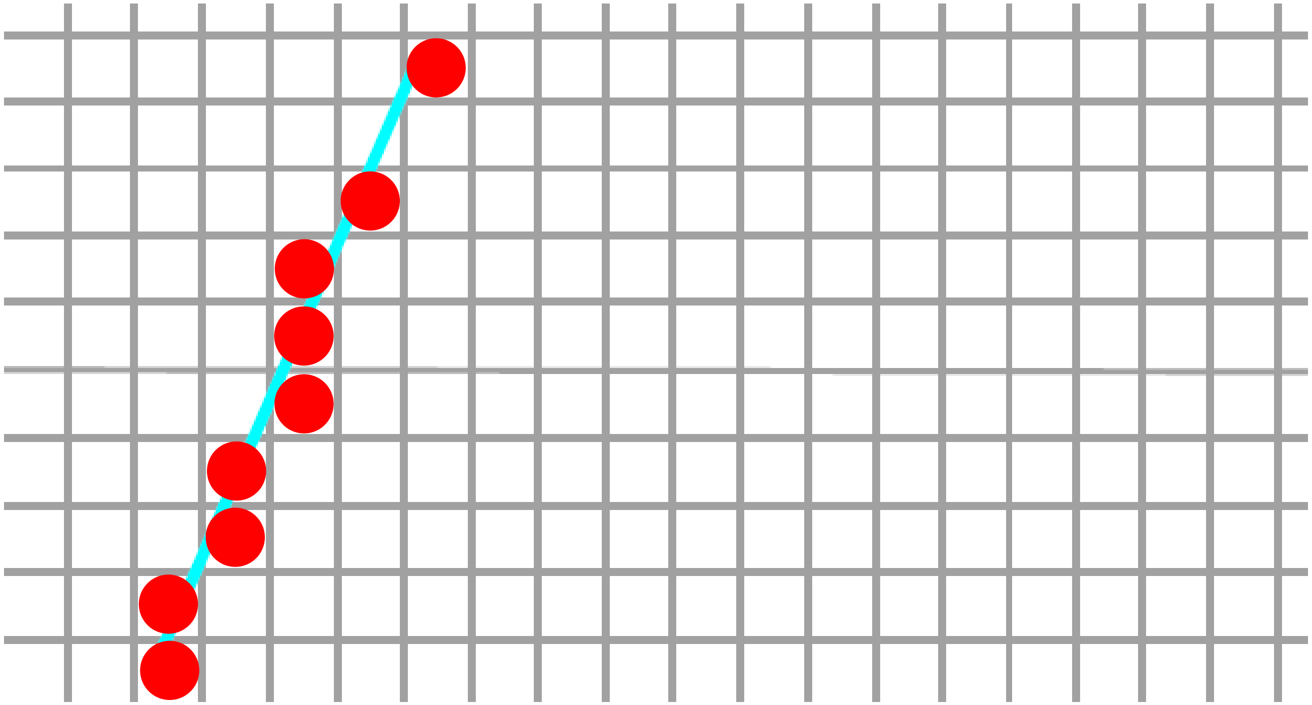
If $\triangle x < \triangle y$



y += 1, x += 1/m

Digital Differential Analyzer (DDA)

If $\triangle x < \triangle y$



y += 1, x += 1/m

Digital Differential Analyzer (DDA)

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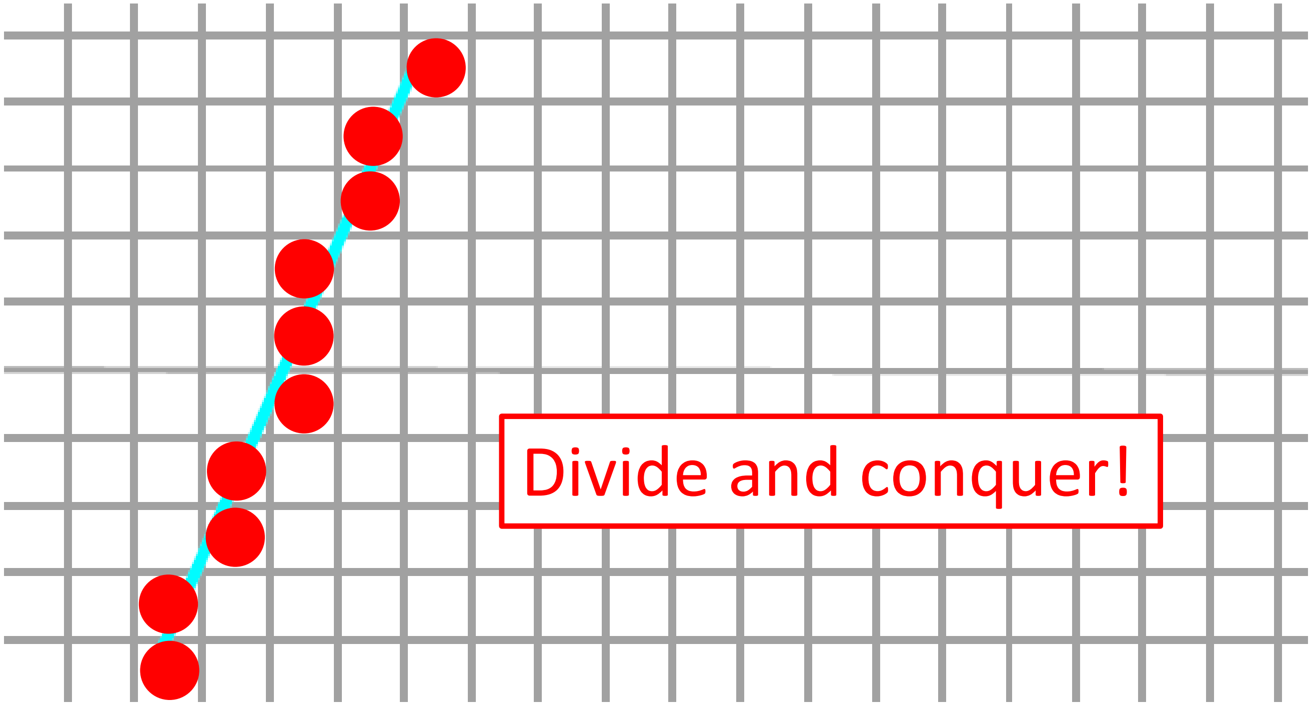


y += 1, x += 1/m

DDA Algorithm

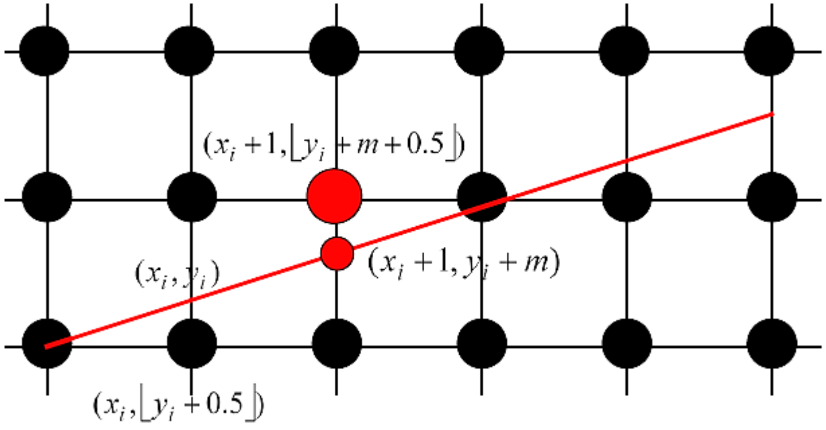
```
#include "device.h"
#include ROUND(a) ((int) (a+0.5))
Void LineDDA( int xa, int ya, int xb, int yb)
{
  int dx =xb-xa, dy=yb-ya, steps, k;
  float xIncrement, yIncrement, x=xa, y=ya;

  if (abs(dx)>abs(dy)) steps=abs(dx);
    else steps=abs(dy);
  xIncrement=dx/(float) steps;
  yIncrement=dx/(float) steps;

  setPixel (ROUND(x), ROUND(y));
  for (k=0;k<steps; k++)
  {  x+=xIncrement; y+=Yincrement; SetPixel (ROUND(x), ROUND(y)); }
}
```

Bresenham's algorithm (布兰森汉姆算法)

- Introduced in 1967 by **J. Bresenham** of IBM
- Best-fit approximation under some conditions
- In DDA, only y_i is used to compute y_i+1, the information for selecting the pixel is neglected
- Bresenham algorithm employs the information to constrain the position of the next pixel



Notations

- The line segment is from (x_0, y_0) to (x_1, y_1)
- Denote $\Delta x = x_1 - x_0 > 0, \Delta y = y_1 - y_0 > 0 \quad m = \Delta y / \Delta x$
- Assume that slope $|m| \leq 1$
- Like DDA algorithm, Bresenham Algorithm also starts from $x = x_0$ and increases x coordinate by 1 each time
- Suppose the i-th point is (x_i, y_i)
- Then the next point can only be one of the following two

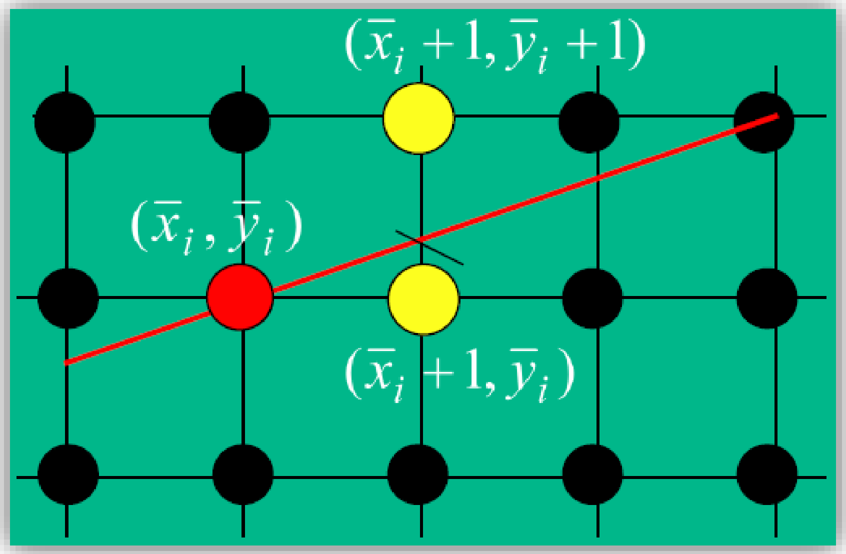
$(\overline{x}_i + 1, \overline{y}_i)$ $(\overline{x}_i + 1, \overline{y}_i + 1)$

Criteria(判别标准)

- We will choose one which distance to the following intersection is shorter

$$x_{i+1} = x_i + 1$$

$$y_{i+1} = mx_{i+1} + B$$
$$= m(x_i + 1) + B.$$

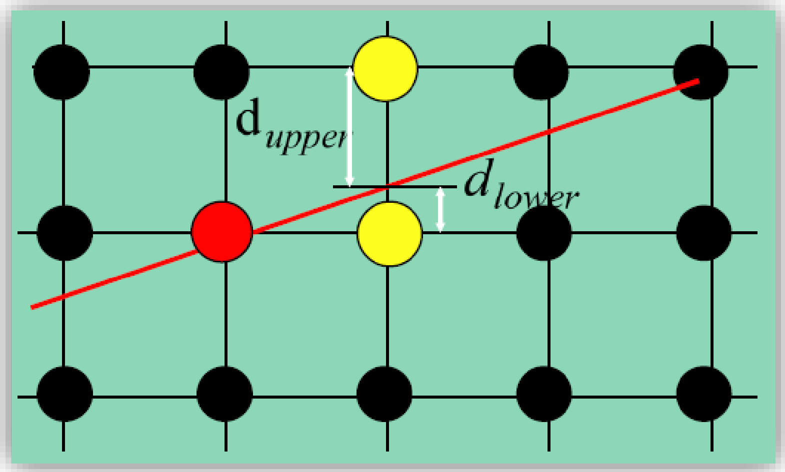


Computation of Criteria

- The distances are respectively

$$d_{upper} = \bar{y}_i + 1 - y_{i+1}$$
$$= \bar{y}_i + 1 - mx_{i+1} - B$$

$$d_{lower} = y_{i+1} - \bar{y}_i$$
$$= mx_{i+1} + B - \bar{y}_i$$



显然：如果 $d_{lower} - d_{upper} > 0$ 则应取右上方的点；如果 $d_{lower} - d_{upper} < 0$ 则应取右边的点； $d_{lower} - d_{upper} = 0$ 可任取，如取右边点。

Computation of Criteria

$$d_{lower} - d_{upper} = m(x_i + 1) + B - \bar{y}_i - (\bar{y}_i + 1 - m(x_i + 1) - B)$$

$$= 2m(x_i + 1) - 2\bar{y}_i + 2B - 1$$

division operation

- **It has the same sign with**

$$p_i = \Delta x \bullet (d_{lower} - d_{upper}) = 2\Delta y \bullet (x_i + 1) - 2\Delta x \bullet \bar{y}_i + (2B - 1)\Delta x$$

$$= 2\Delta y \bullet x_i - 2\Delta x \bullet \bar{y}_i + (2B - 1)\Delta x + 2\Delta y$$

$$= 2\Delta y \bullet x_i - 2\Delta x \bullet \bar{y}_i + c$$

where

$$\Delta x = x_1 - x_0, \Delta y = y_1 - y_0, \quad m = \Delta y / \Delta x$$

$$c = (2B - 1)\Delta x + 2\Delta y$$

Restatement of the Criteria

- If $p_i \succ 0$, then $(\bar{x}_i + 1, \bar{y}_i + 1)$ is selected

  If $p_i \prec 0$, then $(\bar{x}_i + 1, \bar{y}_i)$ is selected

  If $p_i = 0$, *arbitrary one*

- **Can we simplify the computation of p_i ?**

$$
\begin{aligned}
p_0 &= 2\Delta y \bullet x_0 - 2\boxed{\Delta x \bullet \bar{y}_0} + (2B-1)\Delta x + 2\Delta y \\
&= 2\Delta y \bullet x_0 - 2\boxed{(\Delta y \bullet x_0 + B \bullet \Delta x)} + (2B-1)\Delta x + 2\Delta y \\
&= 2\Delta y - \Delta x
\end{aligned}
$$

$$y_{i+1} = mx_{i+1} + B$$

Recursive for computation of p_i

- As

$$p_{i+1} - p_i = (2\Delta y \bullet x_{i+1} - 2\Delta x \bullet \bar{y}_{i+1} + c) - (2\Delta y \bullet x_i - 2\Delta x \bullet \bar{y}_i + c)$$
$$= 2\Delta y - 2\Delta x(\bar{y}_{i+1} - \bar{y}_i)$$

- **If** $p_i \le 0$ **then** $\bar{y}_{i+1} - \bar{y}_i = 0$ **therefore**

$$p_{i+1} = p_i + 2\Delta y$$

- **If** $p_i > 0$ **then** $\bar{y}_{i+1} - \bar{y}_i = 1$ **therefore**

$$p_{i+1} = p_i + 2\Delta y - 2\Delta x$$

Summary of Bresenham Algorithm

- **draw** (x_0, y_0)
- **Calculate** Δx, Δy, $2\Delta y$, $2\Delta y - 2\Delta x$, $p_0 = 2\Delta y - \Delta x$
- **If** $p_i \leq 0$ **draw** $(x_{i+1}, \bar{y}_{i+1}) = (x_i + 1, \bar{y}_i)$

  and compute $p_{i+1} = p_i + 2\Delta y$

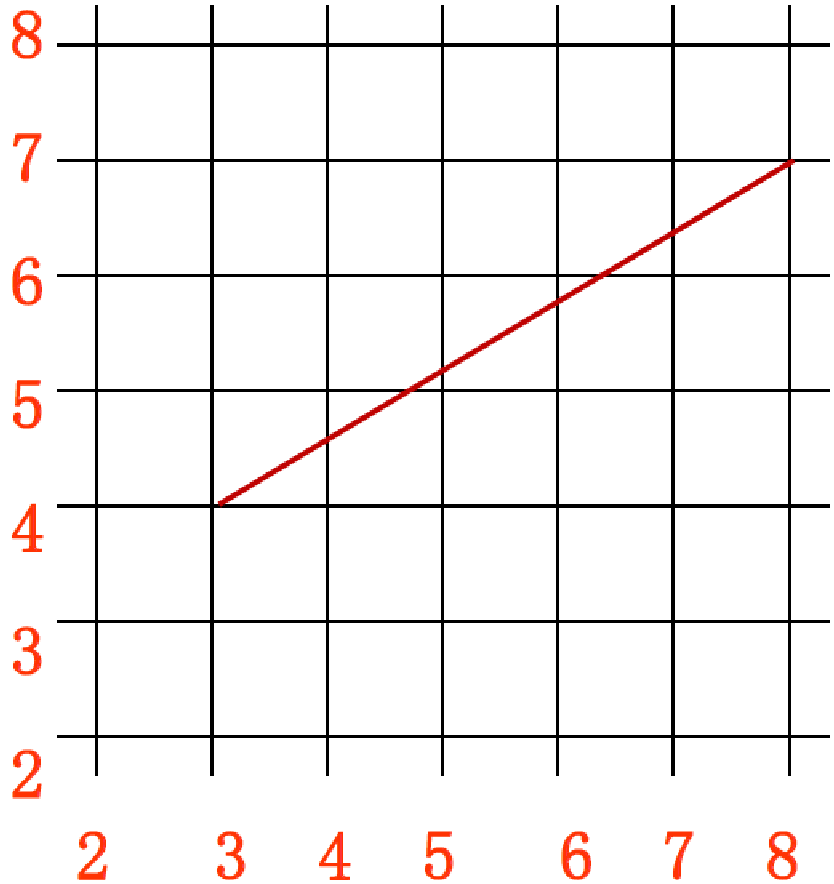
- **If** $p_i > 0$ **draw** $(x_{i+1}, \bar{y}_{i+1}) = (x_i + 1, \bar{y}_i + 1)$

  and compute $p_{i+1} = p_i + 2\Delta y - 2\Delta x$

- **Repeat the last two steps**

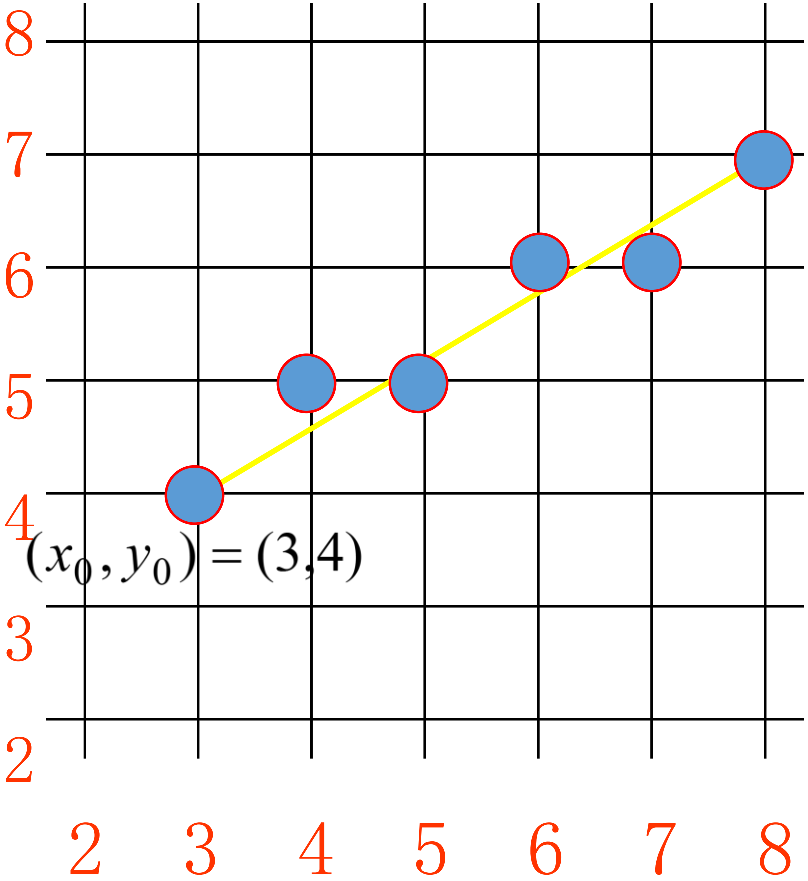
Example

- Draw line segment (3,4)-(8,7)



(Continued)

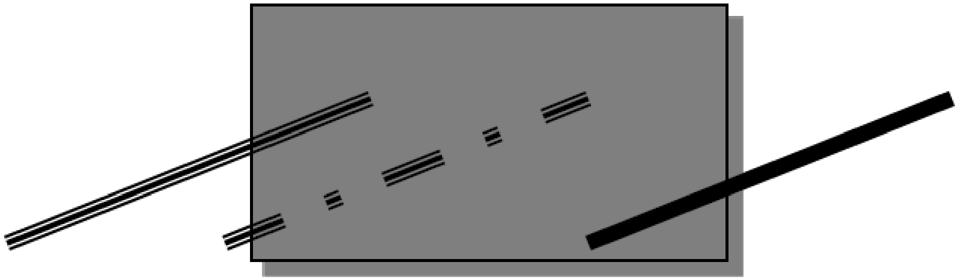
k	p_k	(x_{k+1}, y_{k+1})
0	**1**	**(4,5)**
1	**-3**	**(5,5)**
2	**3**	**(6,6)**
3	**-1**	**(7,6)**
4	**5**	**(8,7)**

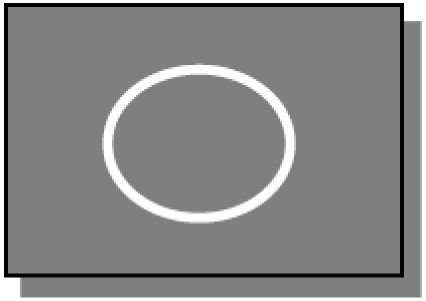


注： $p_0 = 2\Delta y - \Delta x$ $p_{i+1} = p_i + 2\Delta y$ $p_{i+1} = p_i + 2\Delta y - 2\Delta x$

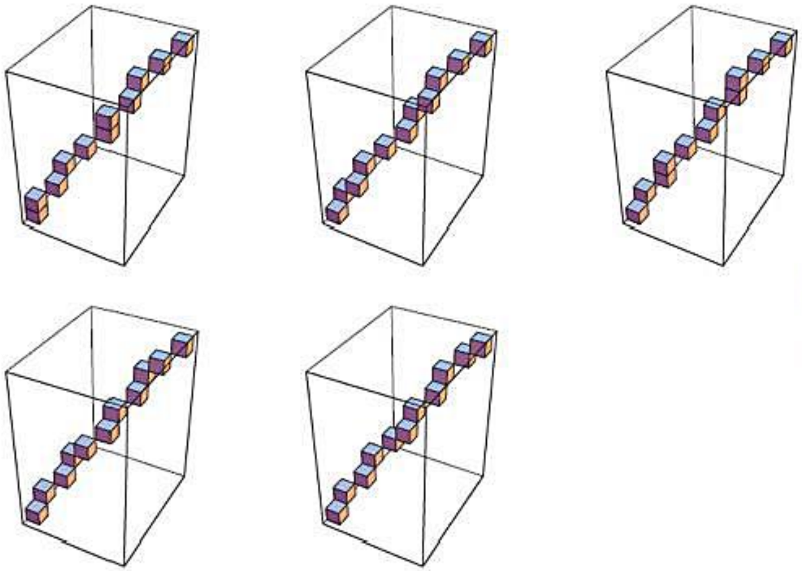
More Raster Line Issues

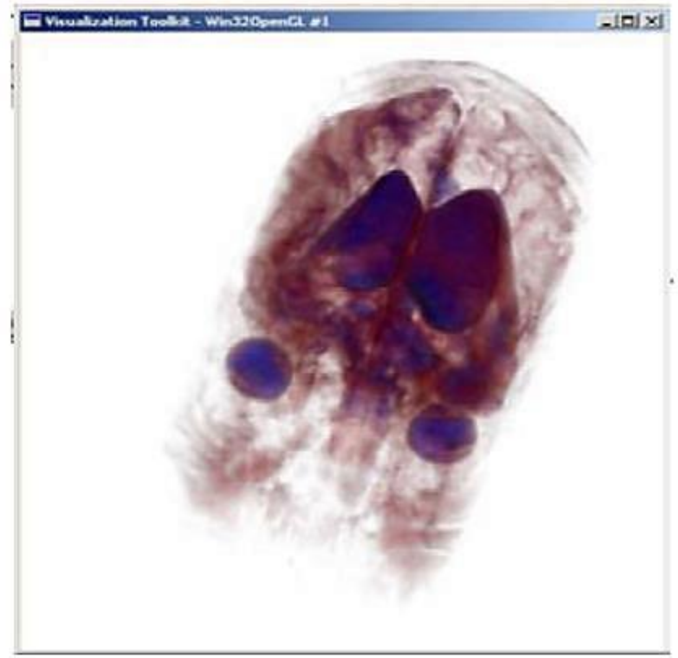
- The coordinates of endpoints are not integer
- Generalize to draw other primitives: circles, ellipsoids
- Line pattern and thickness?





3D Bresenham algorithm





Computer Graphics @ ZJU

Hongxin Zhang, 2014

What Makes a Good Line?

- Not too jaggy
- Uniform thickness of lines at different angles
- Symmetry, Line(P,Q) = Line(Q,P)

- A good line algorithm should be fast.

Line Attributes

- line width
- dash patterns
- end caps: butt, round, square

Line Attributes

- Joins: round, bevel, miter

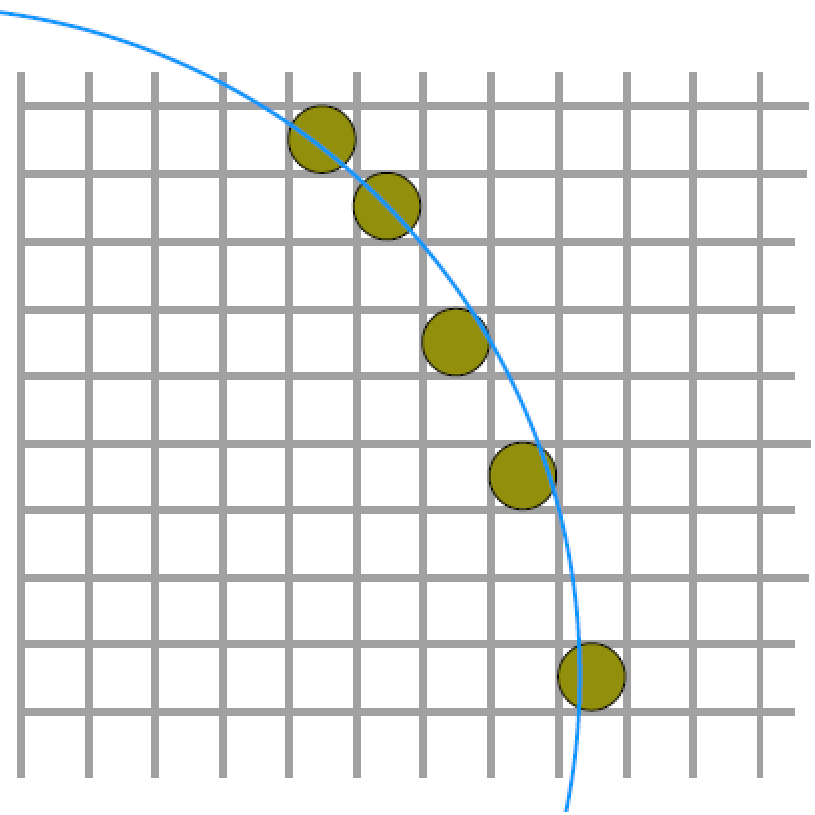
Scan conversion of circles

A circle with center (x_c, y_c) and radius r:

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

orthogonal coordinate

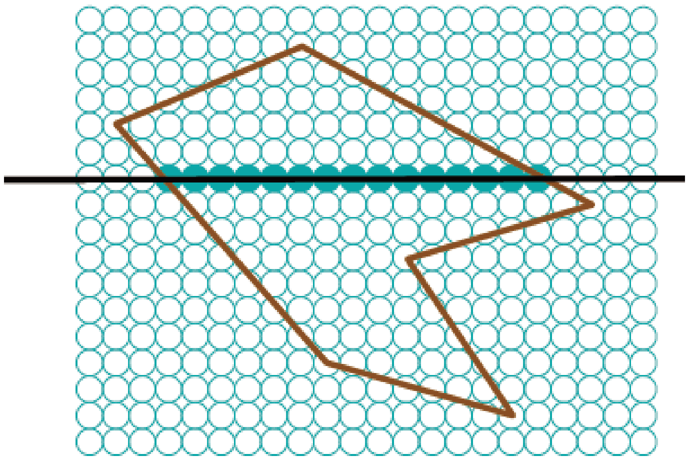
$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$$



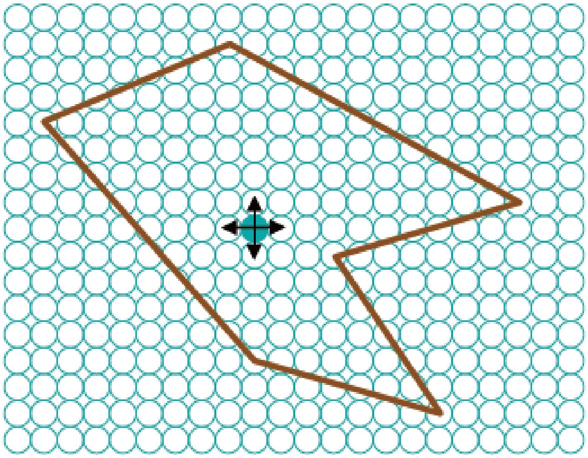
Polygon Rasterization

Takes shapes like triangles and determines which pixels to set

1. Polygon scan-conversion
   - sweep the polygon by scan line, set the pixels whose center is inside the polygon for each scan line

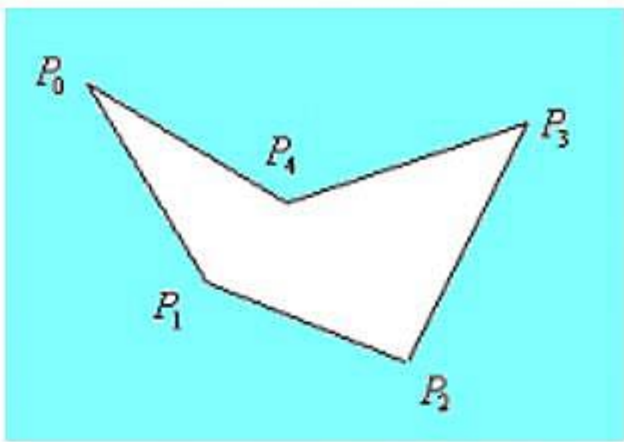


2. Polygon fill
   - select a pixel inside the polygon
   - grow outward until the whole polygon is filled

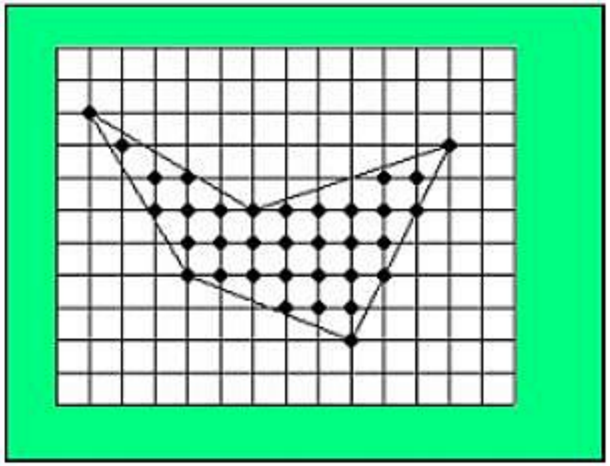


Scan conversion of polygon

- Polygon representation



By vertex



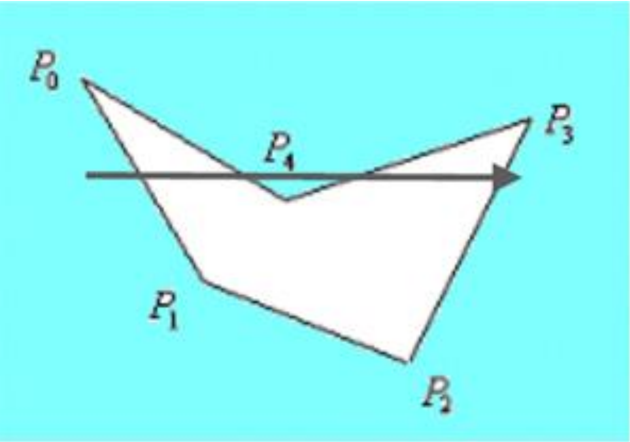
By lattice

- Polygon filling:

  vertex representation → lattice representation

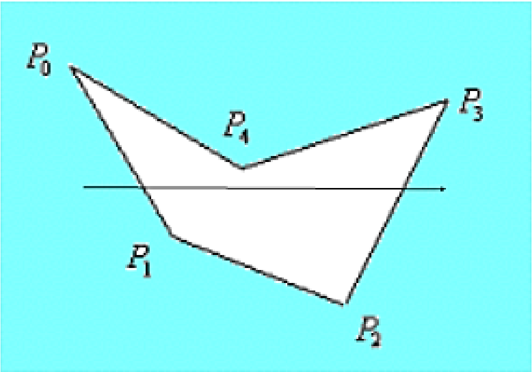
Polygon filling

- fill a polygonal area --> test every pixel in the raster to see if it lies inside the polygon.

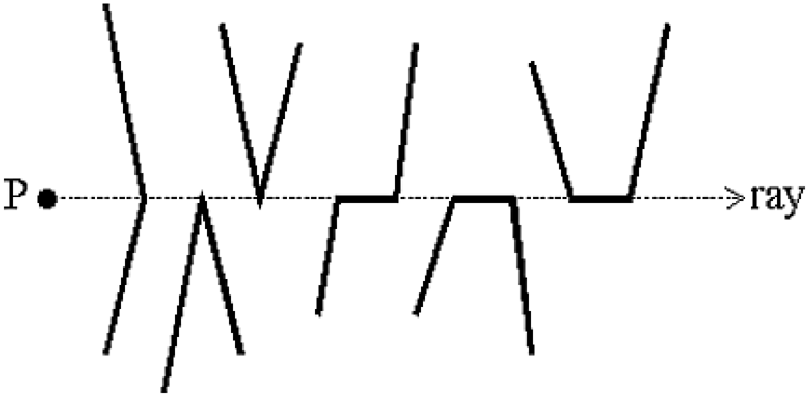


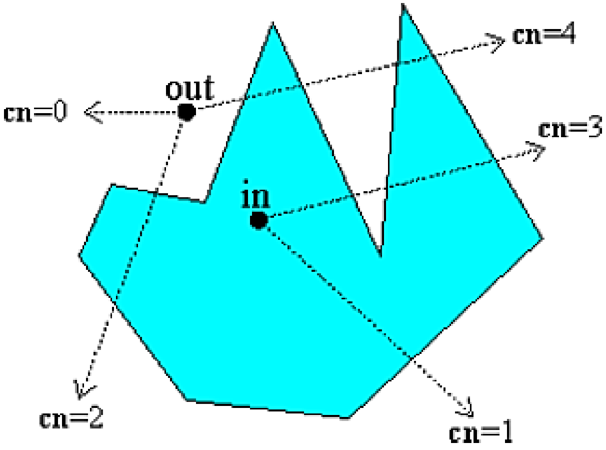
even-odd test

Inside Check



even-odd test





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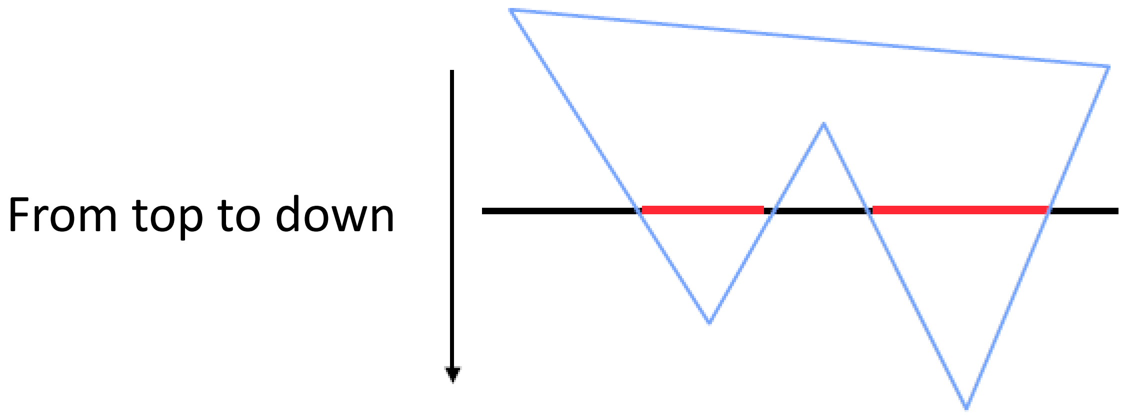
Scan-line Methods

- Makes use of the coherence properties
  - Spatial coherence : Except at the boundary edges, adjacent pixels are likely to have the same characteristics
  - Scan line coherence : Pixels in the adjacent scan lines are likely to have the same characteristics
- Uses intersections between area boundaries and scan lines to identify pixels that are inside the area

Scan Line Method

- Proceeding from left to right the intersections are paired and intervening pixels are set to the specified intensity
- Algorithm
  - Find the intersections of the scan line with all the edges in the polygon
  - Sort the intersections by increasing X-coordinates
  - Fill the pixels between pair of intersections



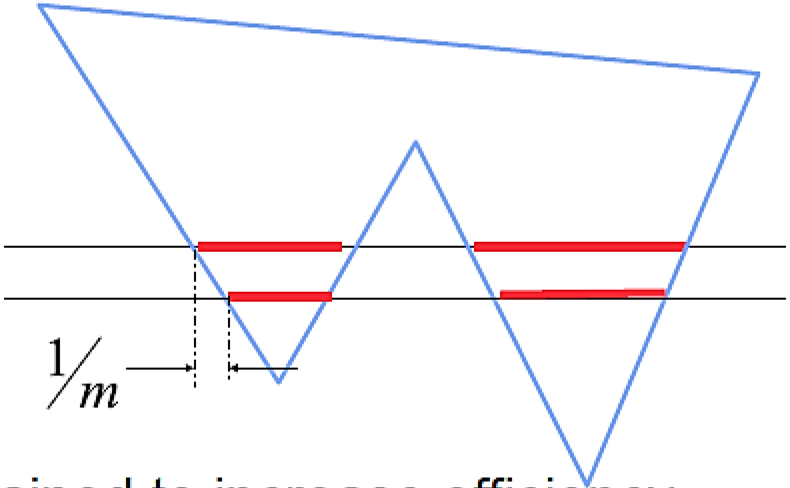
Discussion : How to speed up, or how to avoid calculating intersection

Efficiency Issues Scan-line Methods

- Intersections could be found using edge coherence

  the X-intersection value x_{i+1} of the lower scan line can be computed from the X-intersection value x_i of the preceeding scanline as

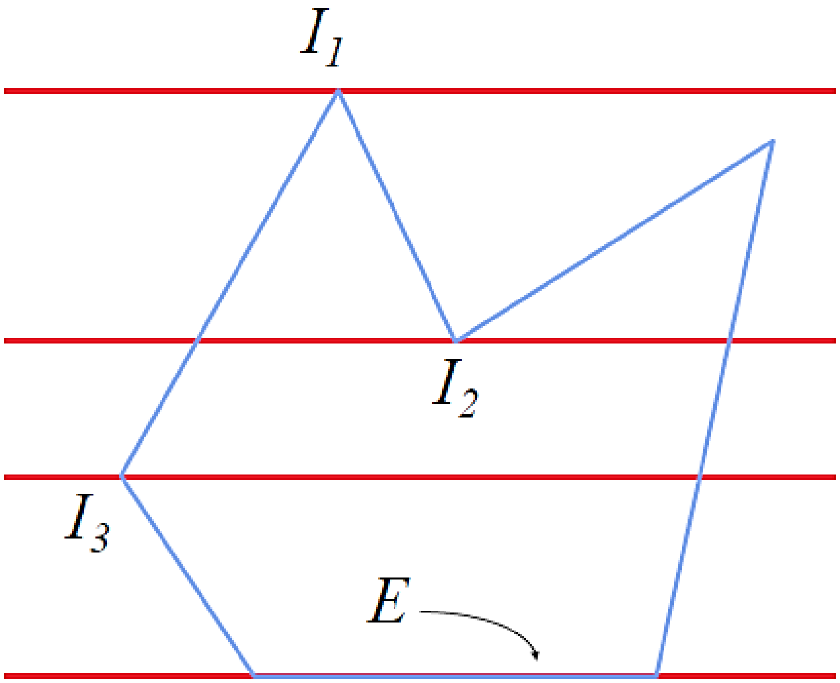
$$x_{i+1} = x_i + 1/m$$

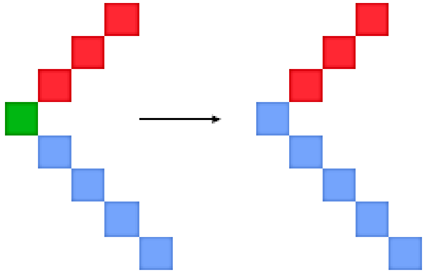


- List of active edges could be maintained to increase efficiency
- Efficiency could be further improved if polygons are convex, much better if they are only triangles

Special cases for Scan-line Methods

- Overall topology should be considered for intersection at the vertices
- Intersections like I_1 and I_2 should be considered as two intersections
- Intersections like I_3 should be considered as one intersection
- Horizontal edges like E need not be considered



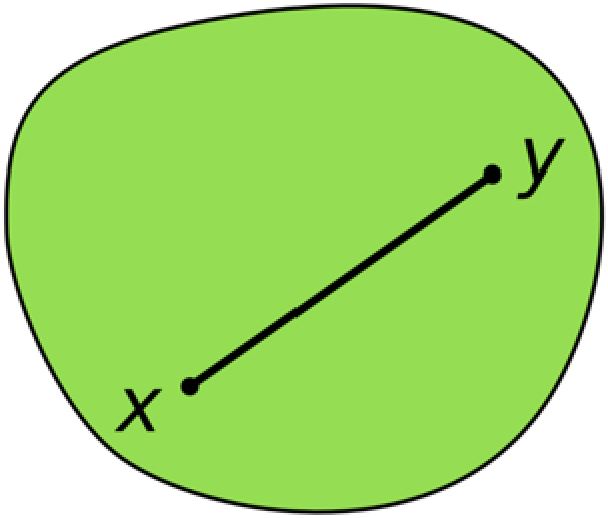


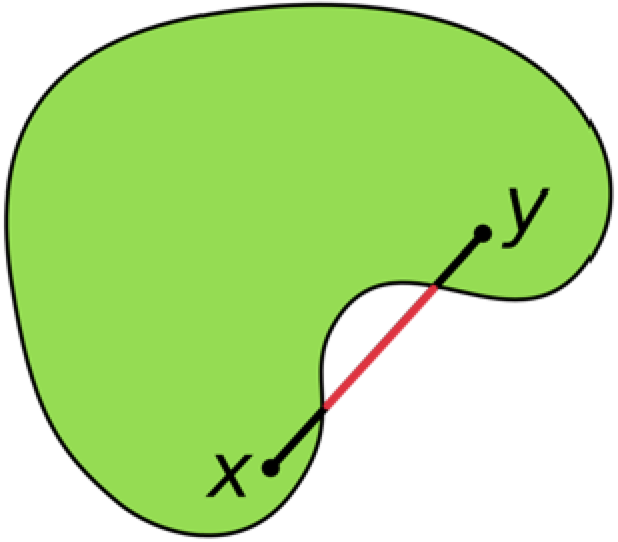
Advantages of Scan Line method

- The algorithm is efficient
- Each pixel is visited only once
- Shading algorithms could be easily integrated with this method to obtain shaded area

- Efficient could be further improved if polygons are convex
- Much better if they are only triangles

What is Convex?



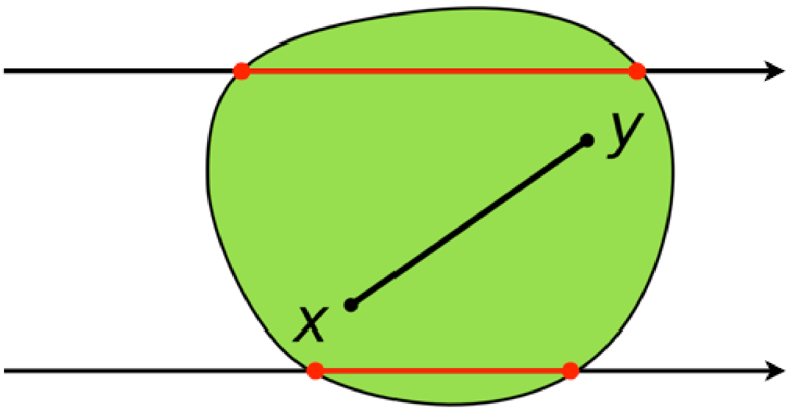


A set C in S is said to be **convex** if, for all x and y in C and all t in the interval [0,1], the point

$$(1-t)\,x + t\,y$$

is in C.

Convex Polygon Rasterization



One in and one out

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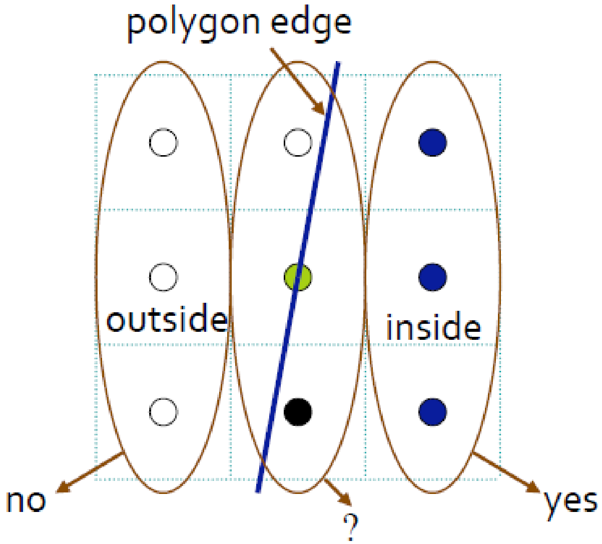
Triangle Rasterization

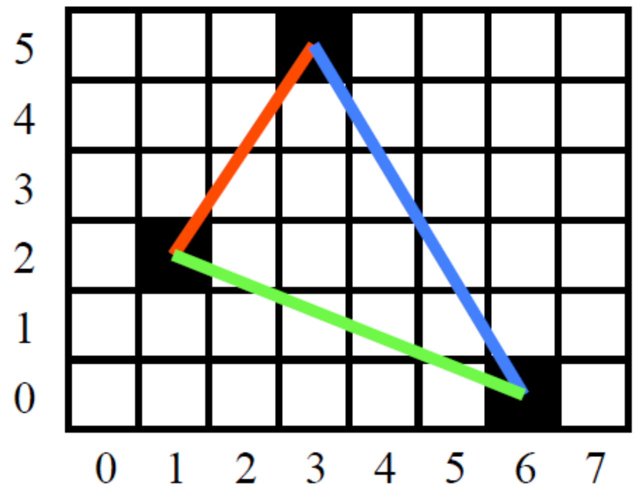
Two questions:

- which pixel to set?
- what color to set each pixel to?

How would you rasterize a triangle?

1. Edge-walking
2. Edge-equation
3. Barycentric-coordinate based

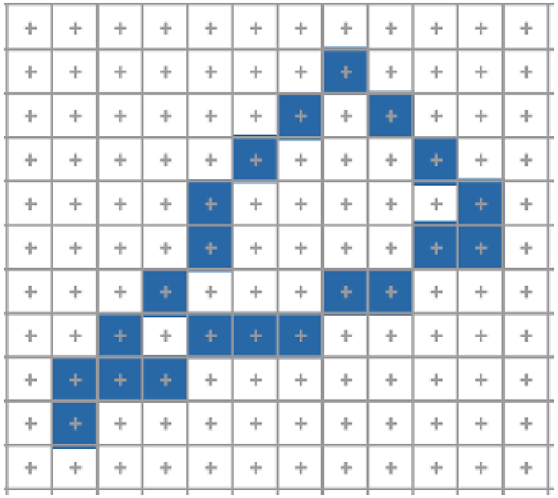


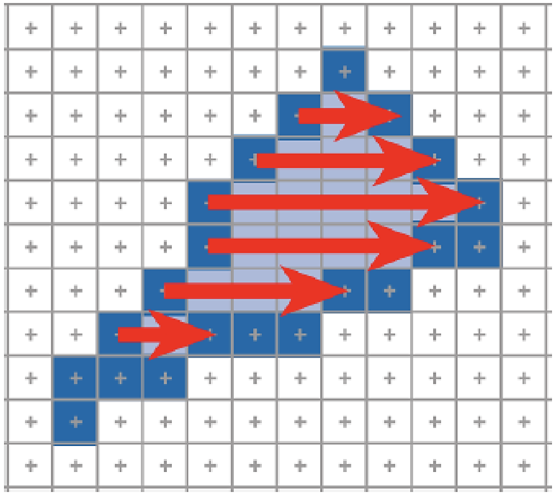


Edge Walking

Idea:

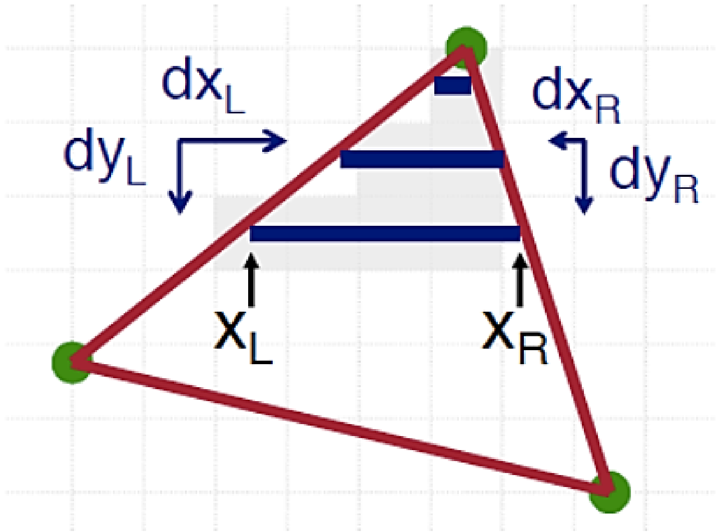
- scan top to bottom in scan-line order
- "walk" edges: use edge slope to update coordinates incrementally
- on each scan-line, scan left to right (horizontal span), setting pixels
- stop when bottom vertex or edge is reached





Edge Walking

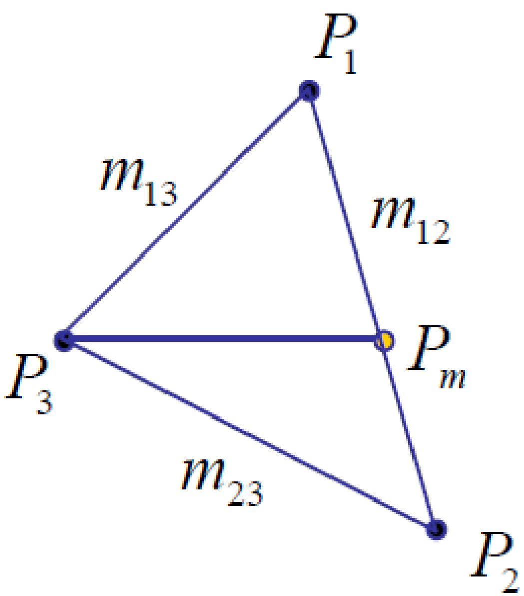
```
void edge_walking(vertices T[3])
{
  for each edge pair of T {
    initialize xL, xR;
    compute dxL/dyL and dxR/dyR;
    for scanline at y {
      for (int x = xL; x <= xR; x++) {
        set_pixel(x, y);
      }
    }
    xL += dxL/dyL;
    xR += dxR/dyR;
  }
}
```



Funkhouser09

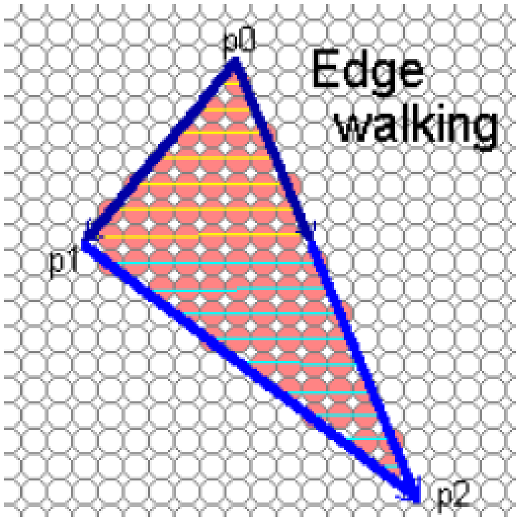
Edge Walking Triangle

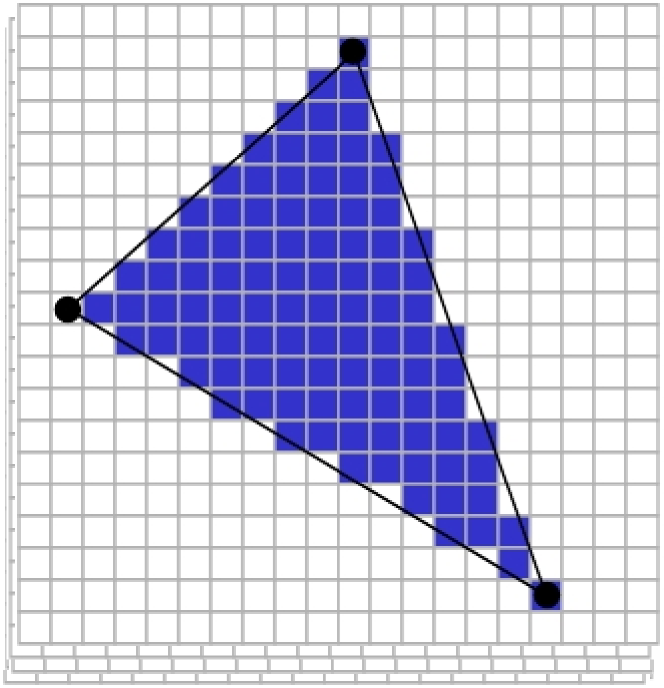
- Split triangles into two "trapezoids" with continuous left and right edges



scanTrapezoid($x_3, x_m, y_3, y_1, \frac{1}{m_{13}}, \frac{1}{m_{12}}$)

scanTrapezoid($x_2, x_2, y_2, y_3, \frac{1}{m_{23}}, \frac{1}{m_{12}}$)



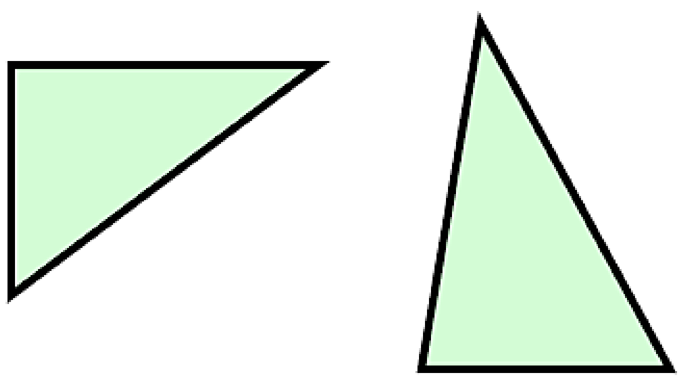


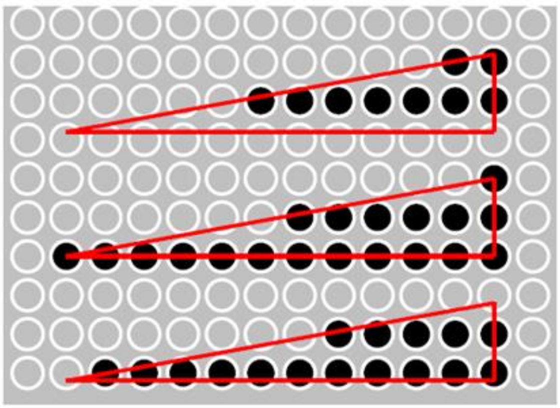
Edge Walking

Advantage: very simple

Disadvantages:

- very serial (one pixel at a time) ⇒ can't parallelize
- inner loop bottleneck if lots of computation per pixel
- special cases will make your life miserable
  - horizontal edges: computing intersection causes divide by 0!

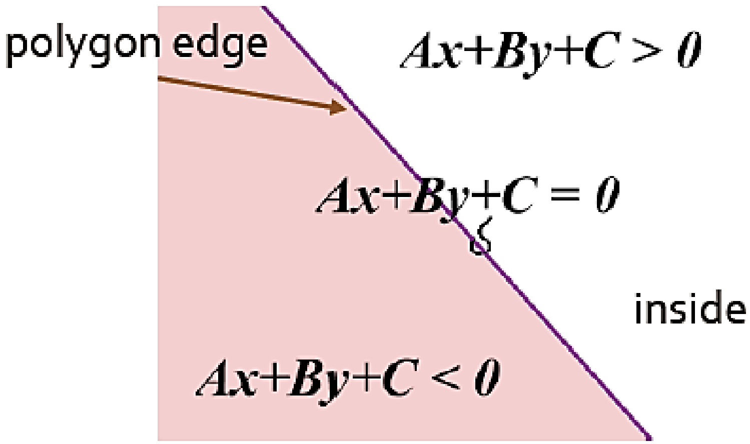




  - sliver: not even a single pixel wide

Edge Equations

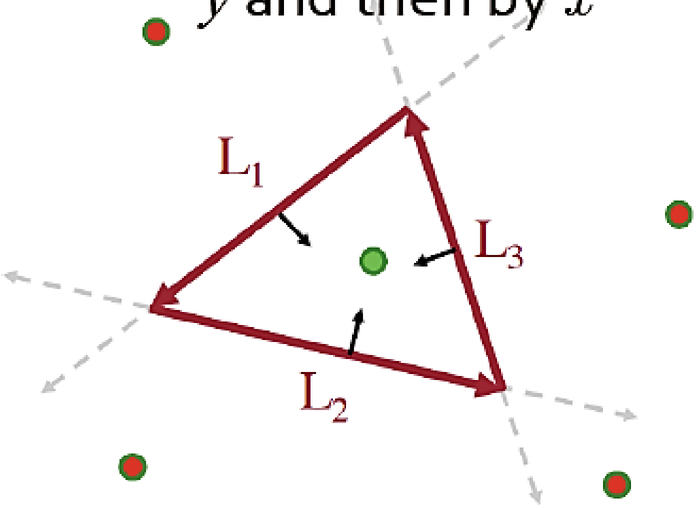
1. compute edge equations from vertices
   - orient edge equations: let negative halfspaces be on the triangle's exterior (multiply by -1 if necessary)
2. scan through each pixel and evaluate against all edge equations
3. set pixel if all three edge equations > 0



Edge Equations

```
void edge_equations(vertices T[3])
{
  bbox b = bound(T);
  foreach pixel(x, y) in b {
    inside = true;
    foreach edge line Li of Tri {
      if (Li.A*x+Li.B*y+Li.C < 0) {
        inside = false;
      }
    }
    if (inside) {
      set_pixel(x, y);
    }
  }
}
```

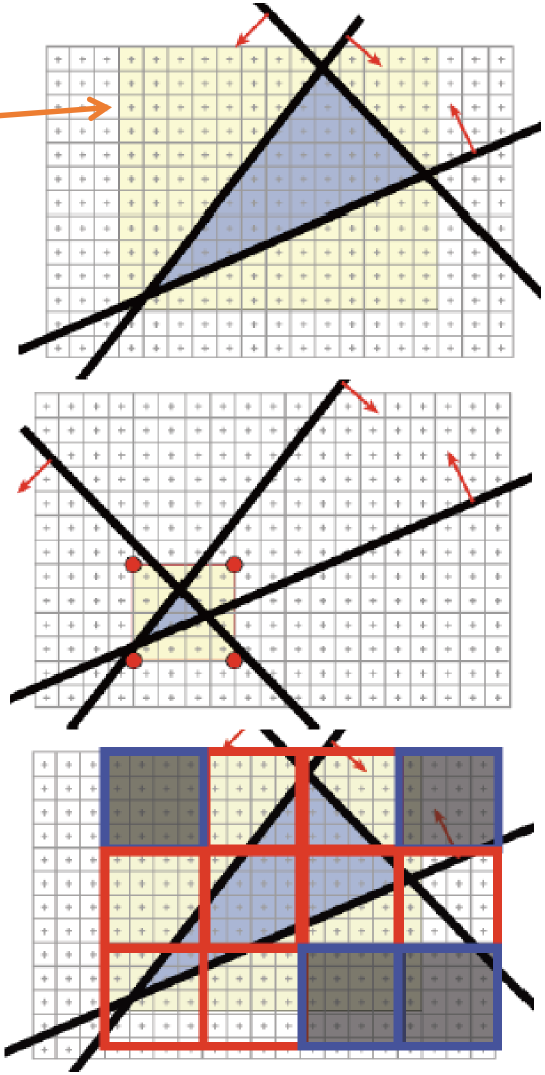
can be rewritten to update the L's incrementally by y and then by x



Edge Equations

Can we reduce #pixels tested?

1. compute a bounding box:
   $x_{min}, y_{min}, x_{max}, y_{max}$ of triangle
2. compute edge equations from vertices
   - orient edge equations: let negative halfspaces be on the triangle's exterior (multiply by -1 if necessary)
   - can be done incrementally per scan line
3. scan through *each* pixel in bounding box and evaluate against all edge equations
4. set pixel if all three edge equations > 0

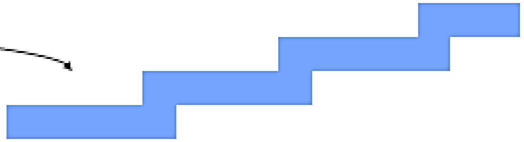


Hierarchical bounding boxes

- how to quickly exclude a bounding box?

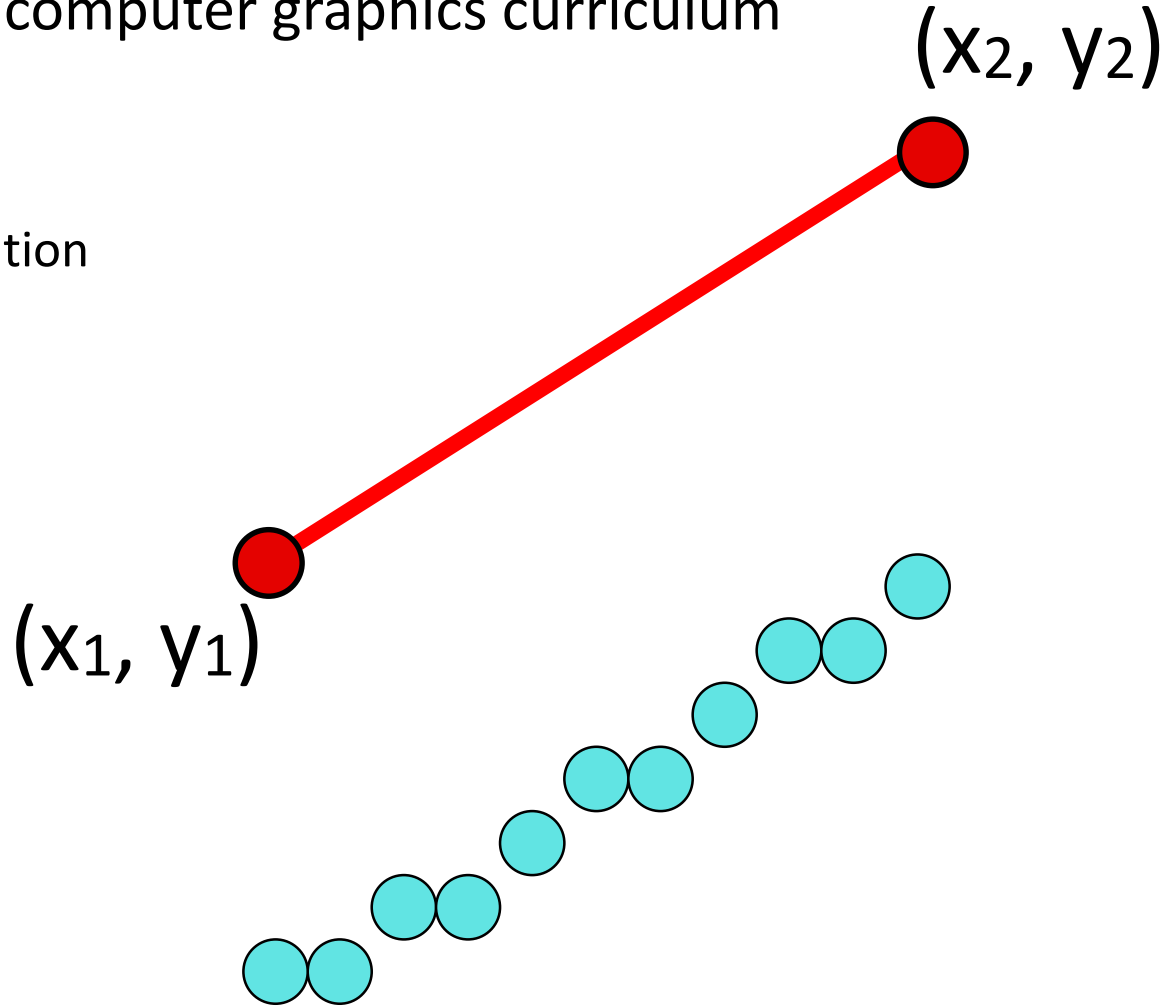
Aliasing

- Aliasing is caused due to the discrete nature of the display device
- Rasterizing primitives is like sampling a continuous signal by a finite set of values (point sampling)
- Information is lost if the rate of sampling is not sufficient. This sampling error is called ***aliasing***.
- Effects of aliasing are
  - Jagged edges
  - Incorrectly rendered fine details
  - Small objects might miss



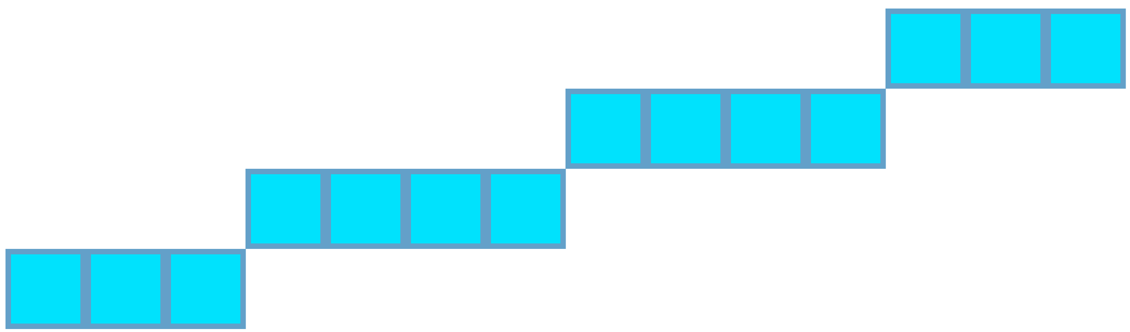
Aliasing

- A classic part of the computer graphics curriculum
- Input:
  - Line segment definition
  - (x1, y1), (x2, y2)
- Output:
  - List of pixels

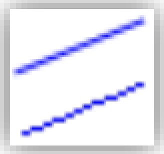


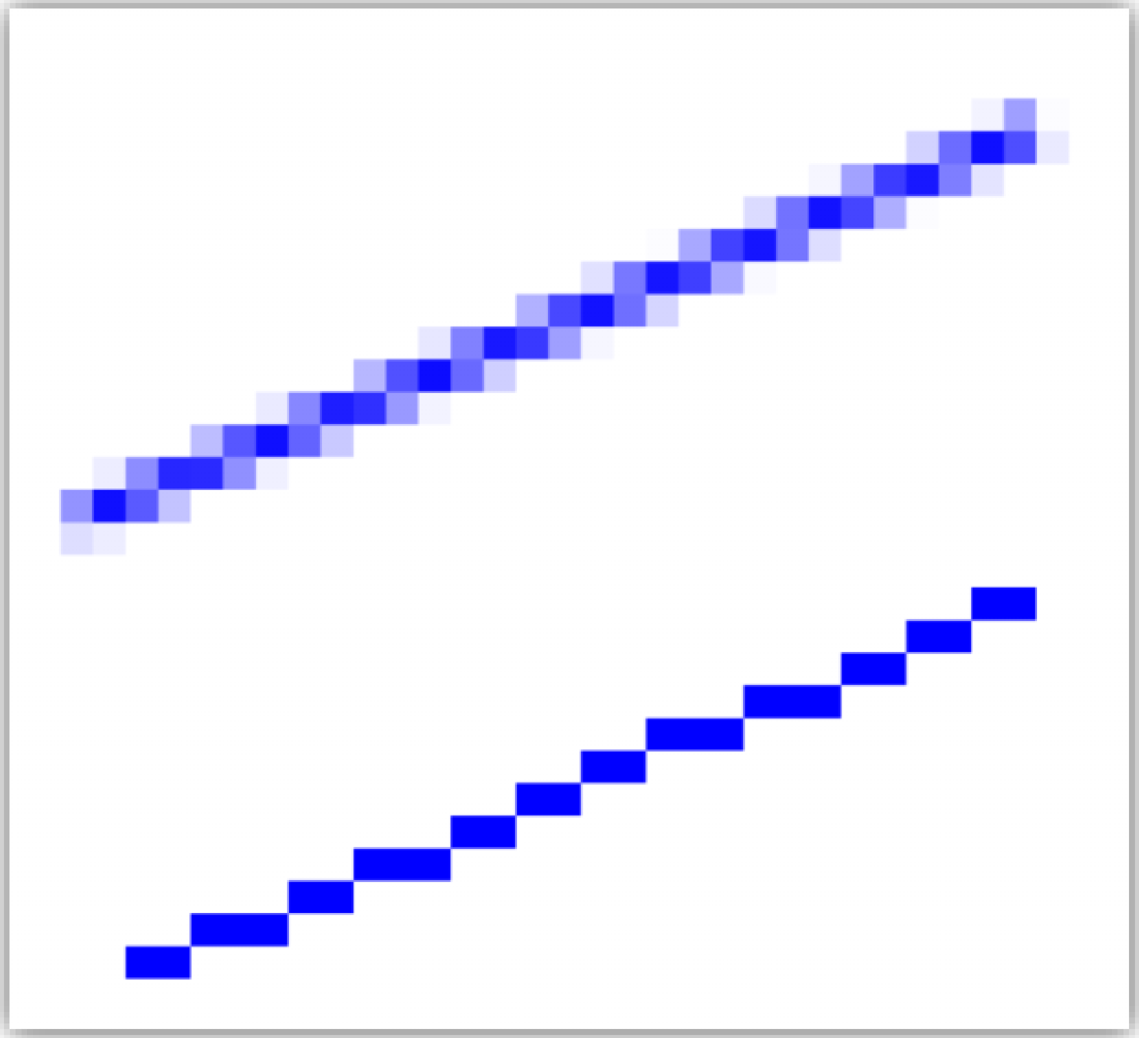
Aliasing

- How Do They Look?
- So now we know how to draw lines
- But they don't look very good:



Antialiasing





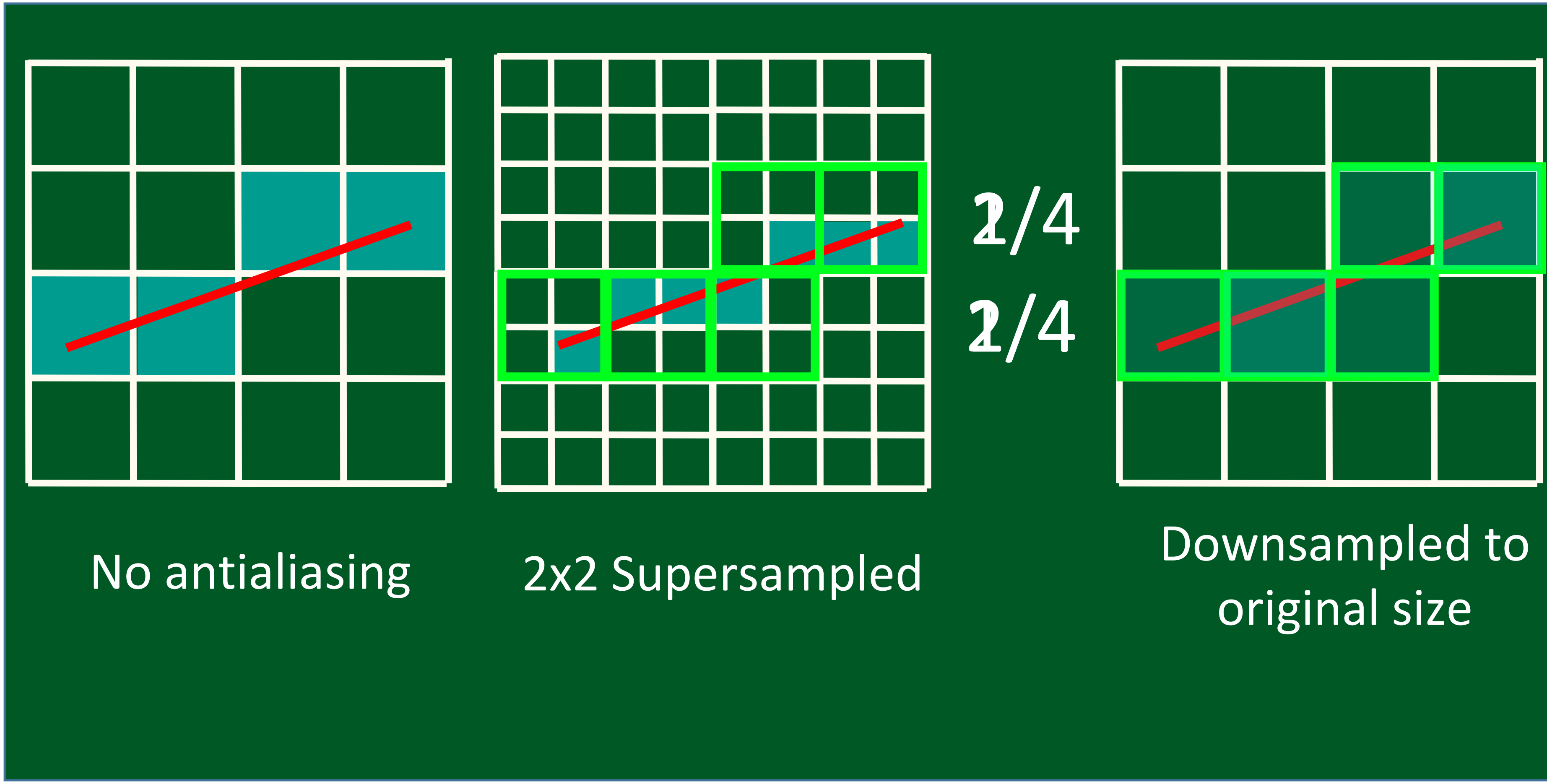
Anti-aliasing

- Application of techniques to reduce/eliminate aliasing artifacts.
- Essentially 3 techniques:
  - Super-sampling vs. filter
    - We discussed a simple averaging filter
  - Compute the fraction of a line that should be applied to a pixel
    - Ratio method
  - Area Simpling

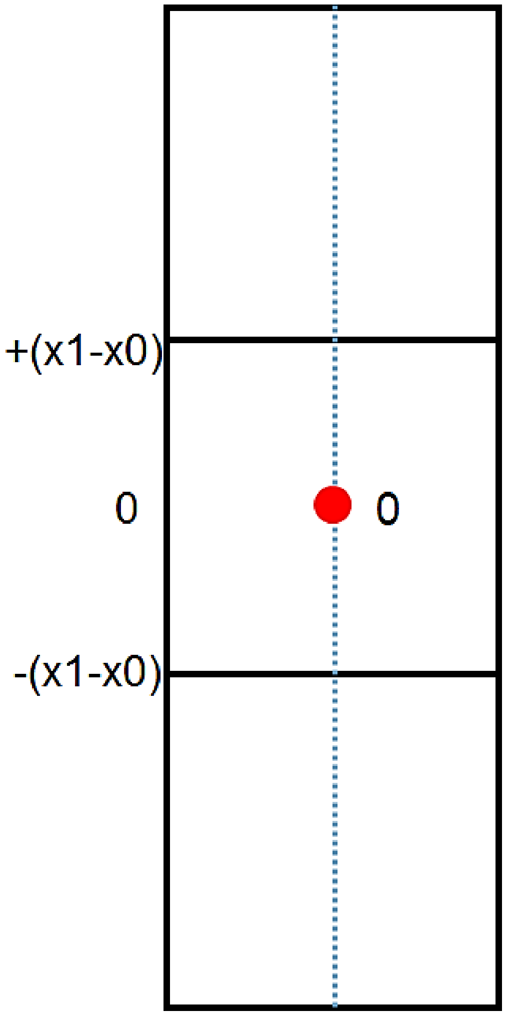
Anti-aliasing: Super-sampling

- Technique:
    1. Create an image 2x (or 4x, or 8x) bigger than the real image
    2. Scale the line endpoints accordingly
    3. Draw the line as before
        - No change to line drawing algorithm
    4. Average each 2x2 (or 4x4, or 8x8) block into a single pixel

Anti-aliasing: Super-sampling



Anti-aliasing: Ratios

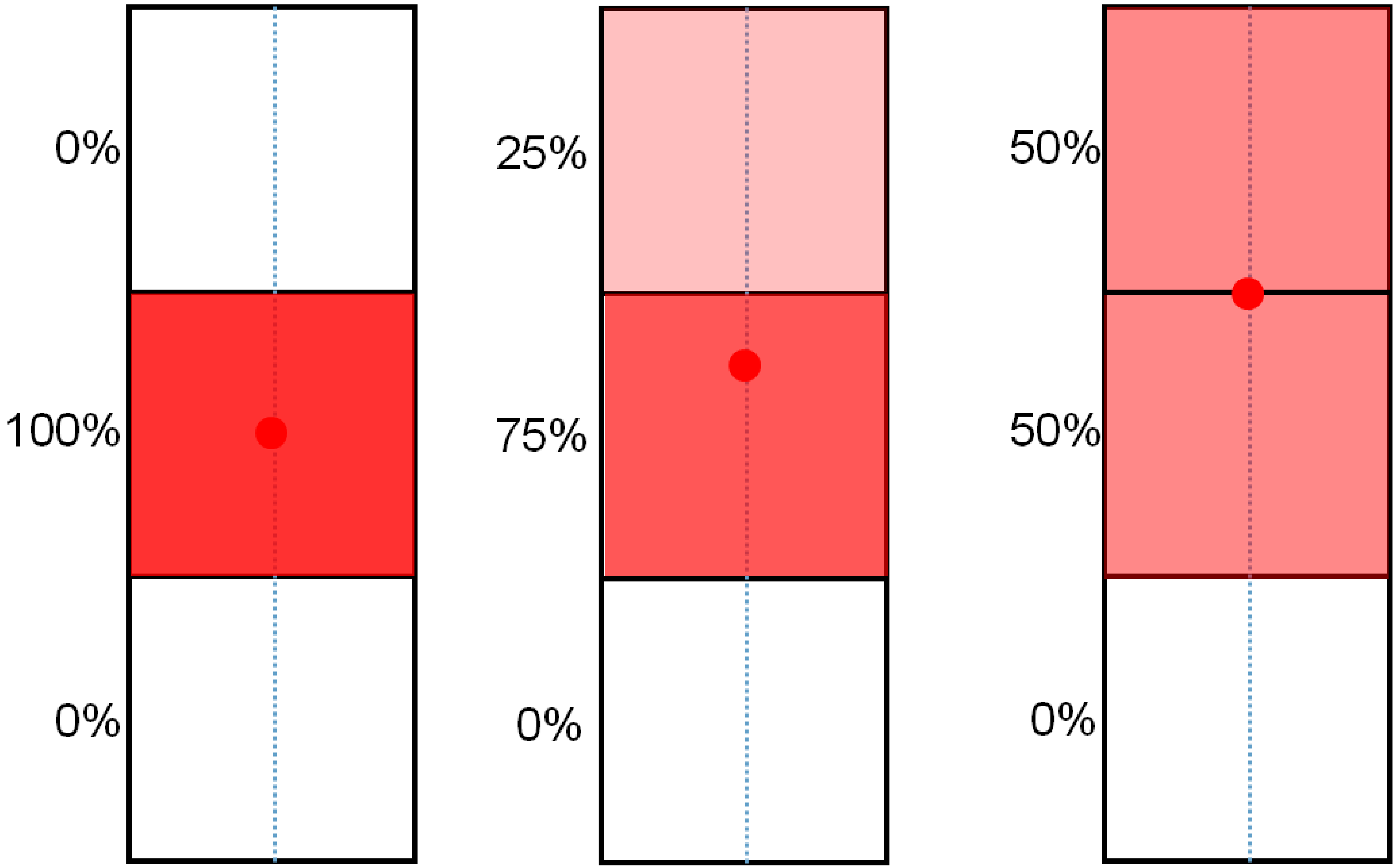


$$\left(.5 * MAX\left(\frac{error}{x_1 - x_0}, 0\right)\right) RGB$$

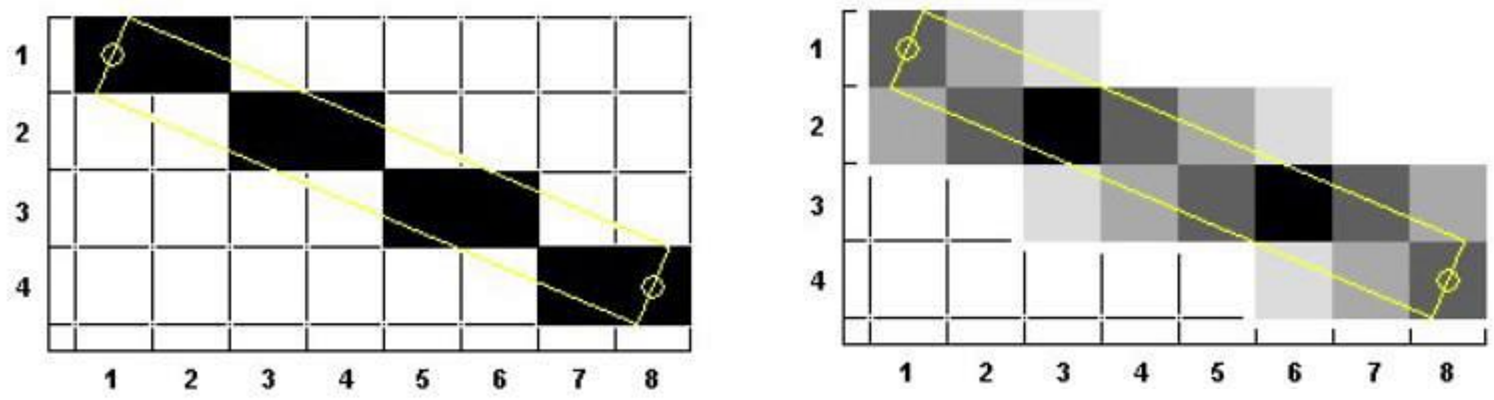
$$\left(1.0 - .5 * abs\left(\frac{error}{x_1 - x_0}\right)\right) RGB$$

$$\left(.5 * MAX\left(\frac{-error}{x_1 - x_0}, 0\right)\right) RGB$$

Anti-aliasing: Ratios



Anti-aliasing (Area Sampling)



- A scan converted primitive occupies finite area on the screen
- Intensity of the boundary pixels is adjusted depending on the percent of the pixel area covered by the primitive. This is called weighted area sampling